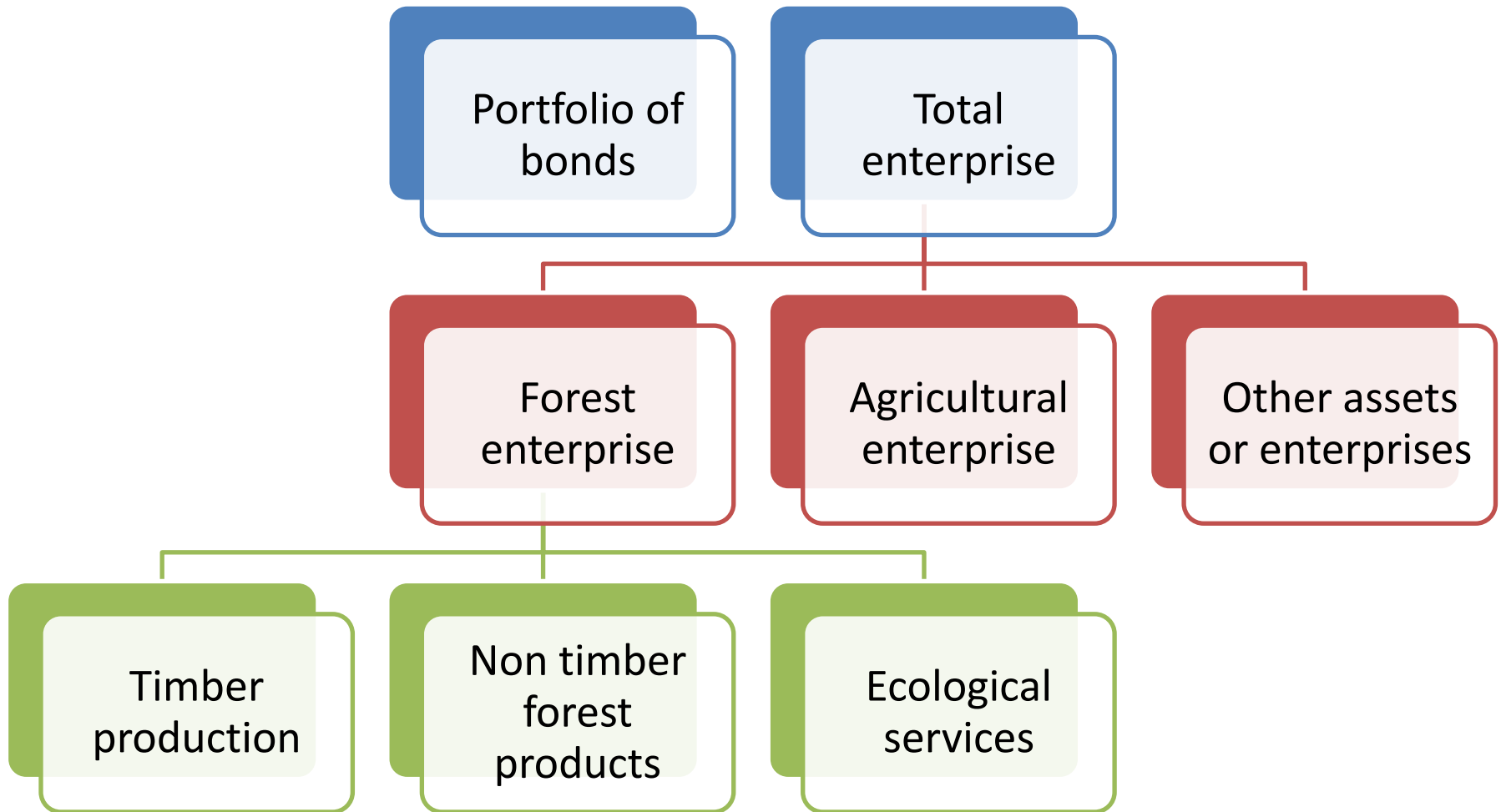


# Simultaneous optimization of rotation periods and tree species composition

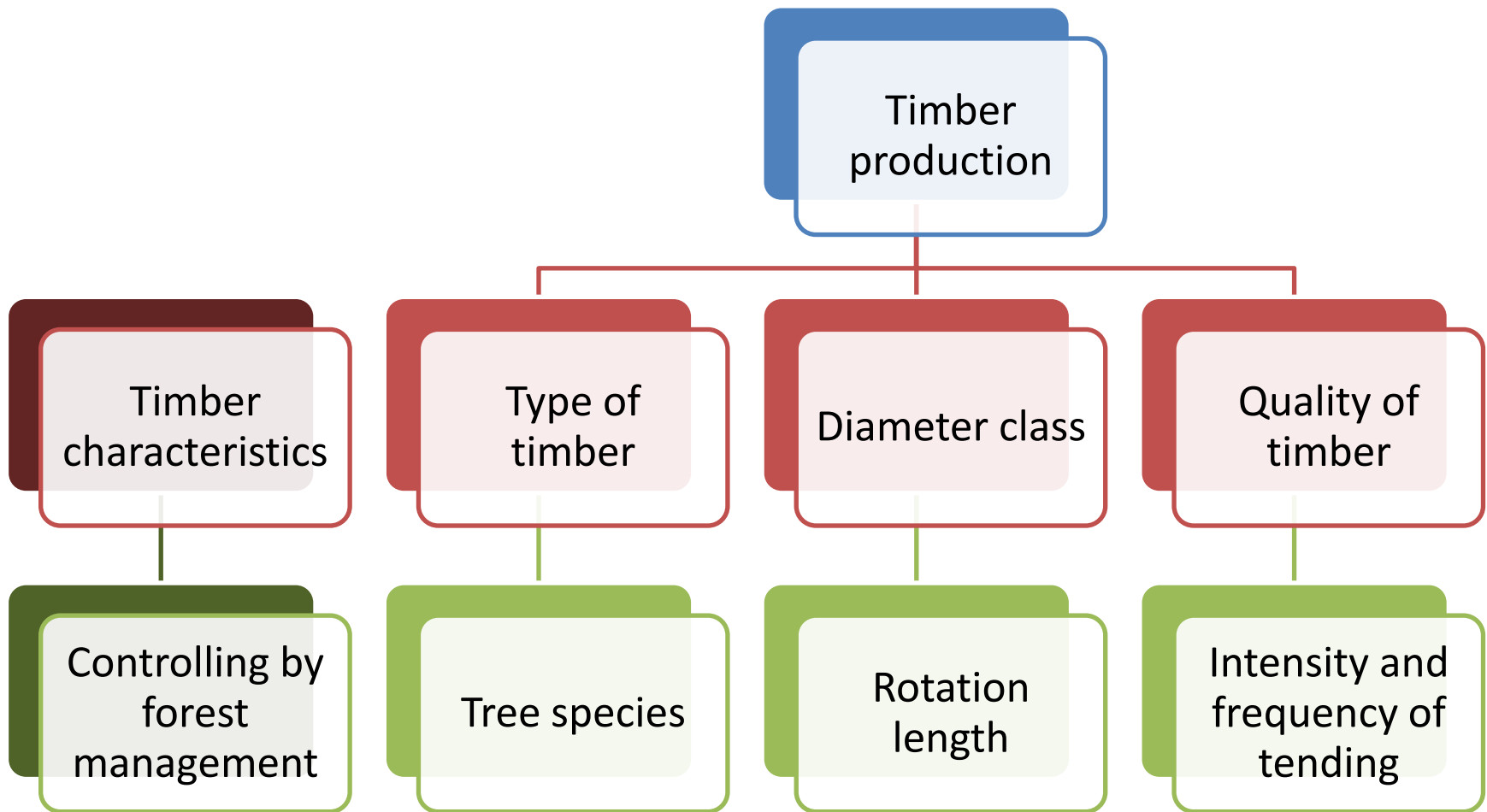
# Question of optimization

- All management decisions in forestry include risks
- Determination of these risks and consider them for planning
- Aim of financial optimization of a forest enterprise is
  - to maximize the expected income or
  - to minimize the risk by diversification
  - by advising the best management decisions

# The diversification effect of forestry



# Diversification in timber production



# Determination of the risk of a composition of timber grades according to Markowitz

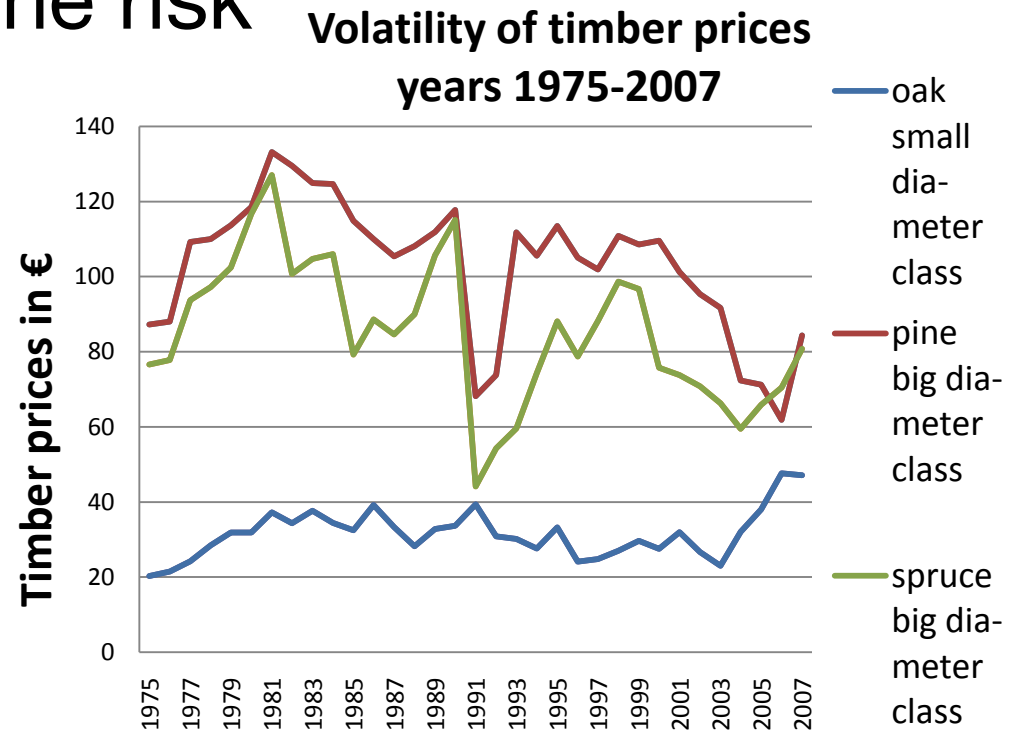
- Correlation coefficients of the timber price statistic describe the risk

- Correlation coefficients:

– pine:spruce +0.80

– oak :spruce +0.05

– oak :pine - 0.07



# Return of a portfolio: Markowitz 1952

$$E_{P_o} = \sum_{i=1}^N a_i \times E_i \quad \sum_{i=1}^N a_i = 1$$

- $E_{P_o}$  expected value of a portfolio
- $E_i$  expected value of investment  $i$
- $a_i$  fraction of investment  $i$  of the total portfolio
- $N$  number of investments ( $a_1$  to  $a_N$ )

# Standard deviation of portfolio return

## Markowitz 1952

$$s_{Po} = \sqrt{\sum_{i \in Po} a_i^2 \times s_i^2 + 2 \times \sum_{i \in Po} \sum_{j \in Po; j > i} a_i \times a_j \times s_i \times s_j \times k_{ij}}$$

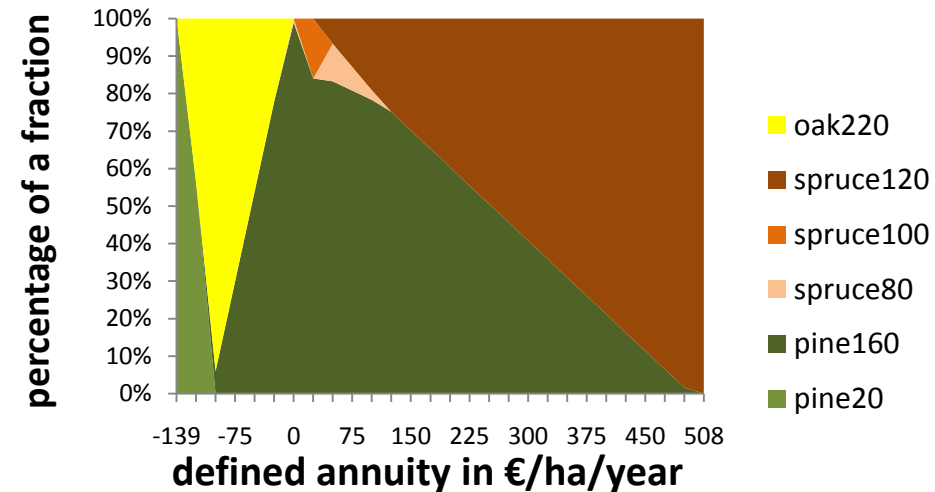
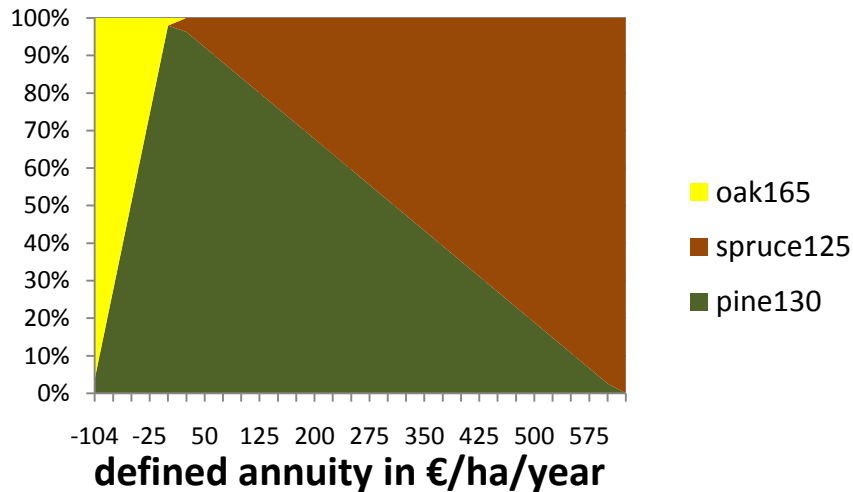
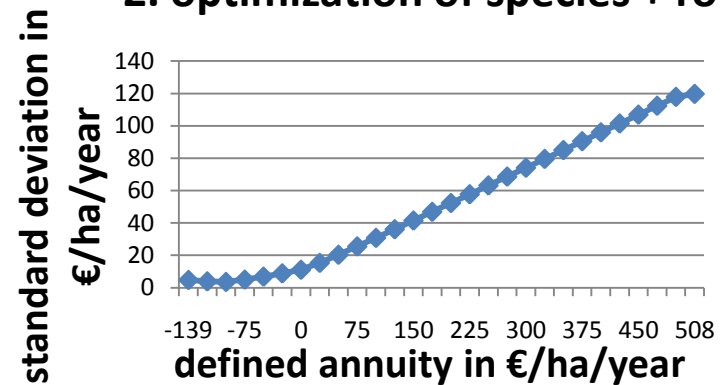
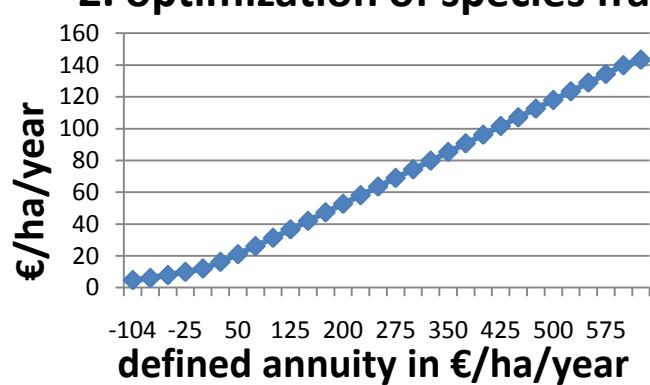
- $s_{Po}$  standard deviation of portfolio return
- $s_i$  standard deviation of portfolio component  $i$
- $k_{ij}$  correlation coefficient of return between the portfolio components  $i$  and  $j$

Example data:			Standard deviation is the root of the sum of:			
	s	a		oak small	pine big	spruce big
oak small	6	0.3	oak small	$0.3 \times 0.3 \times 6 \times 6 \times (+1.00)$	$0.3 \times 0.2 \times 6 \times 18 \times (-0.07)$	$0.3 \times 0.5 \times 6 \times 19 \times (+0.05)$
pine big	18	0.2	pine big	$0.2 \times 0.3 \times 18 \times 6 \times (-0.07)$	$0.2 \times 0.2 \times 18 \times 18 \times (+1.00)$	$0.2 \times 0.5 \times 18 \times 19 \times (+0.80)$
spruce big	19	0.5	spruce big	$0.5 \times 0.3 \times 19 \times 6 \times (+0.05)$	$0.5 \times 0.2 \times 19 \times 18 \times (+0.80)$	$0.5 \times 0.5 \times 19 \times 19 \times (+1.00)$

# Assumptions and results: Several timber grades in each fraction, without natural hazard, $i=2$

1. one rotation length per species
2. optimization of species fractions

1. rotation length in 20 year-steps
2. optimization of species + rotation





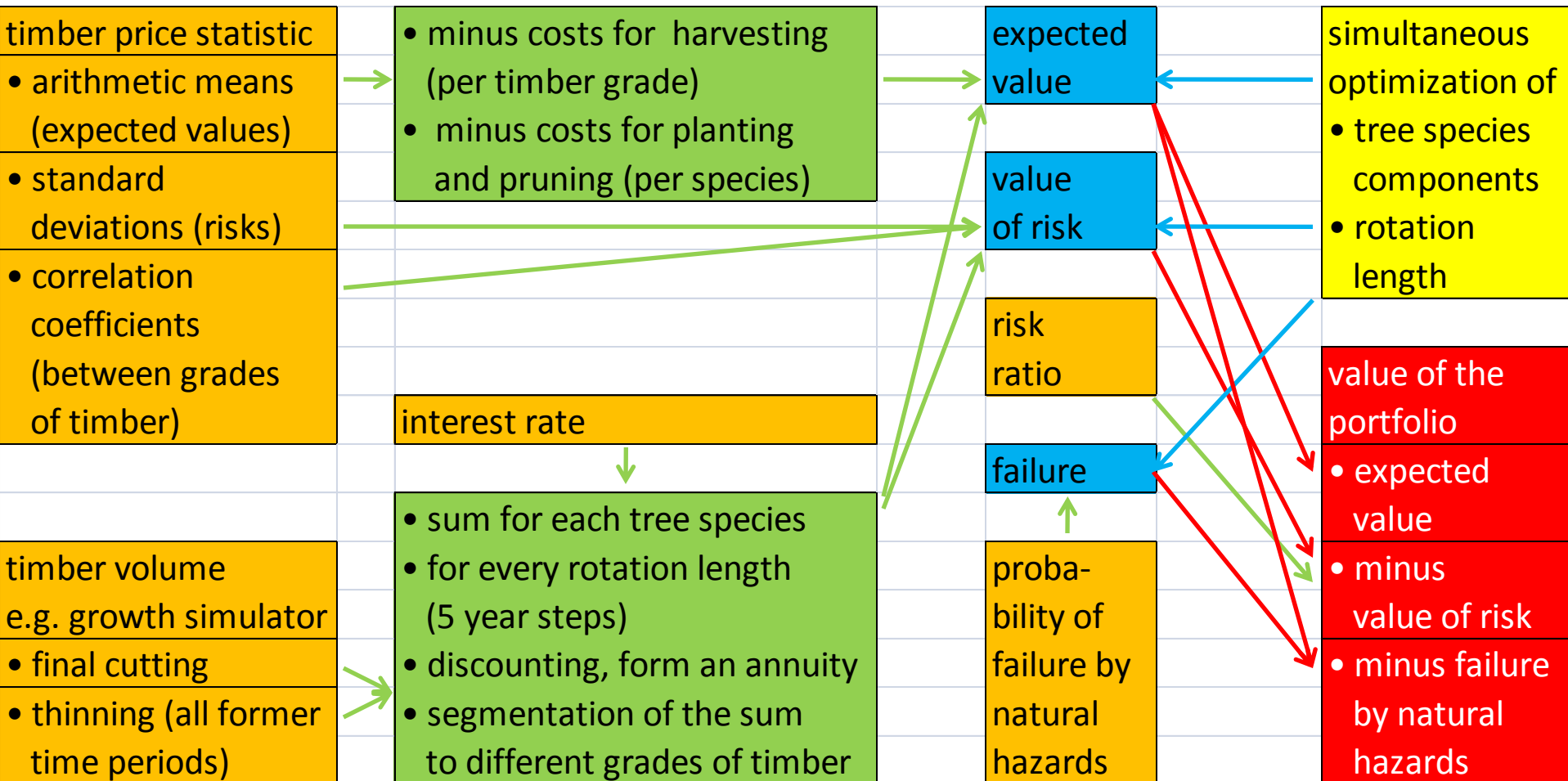
# Conclusion

- The inclusion of risks can help to interpret the financial impacts of planning more realistic
- Diversification of the timber production leads to a better relation between risk and expected income for a forest enterprise
- Simultaneous optimization helps to advise the best management decisions for different factors

# references

- Beinhofer, Bernhard (2009): Zur Anwendung der Portfoliotheorie in der Forstwissenschaft - Finanzielle Optimierungsansätze zur Bewertung von Diversifikationseffekten (dissertation)
- Faustmann, Martin (1849): Berechnung des Werthes, welchen Waldboden, sowie noch nicht haubare Holzbestände für die Waldwirtschaft besitzen. In: Allgemeine Forst- und Jagdzeitung, H. Dezember 1849, S. 442–455.
- Markowitz, Harry (1952): Portfolio selection. In: The Journal of Finance, Jg. 7, H. 1, S. 77–91.

# Steps of the workprocess and necessary data



# Optimization with the certainty equivalent

- Functions of expected value and standard deviation (data from timberprice statistic)

$$CEQ = E(r) - \alpha \times s_e^2 / 2 \quad (\text{Spremann 1996})$$

- $E(r)$  expected value (arithmetic mean)
- $\alpha$  factor of risk aversion
- $\alpha = a / \text{amount of investment}$  (Spremann 1996)
- $a=1$  for normal;  $a=2$  for strong risk aversion
- Failure of natural hazard: only half value

# Different optimization assumptions

- Definition of optimization goal
  - Reach minimal risk for defined annuity
  - Reach maximal annuity for defined risk
- Changeable fractions of optimization
  - One best rotation length of each species
  - Fractions of defined rotation length per species
- Consideration of timber grades per fraction
  - Several timber grades in one species
  - Only one timber grade (then only final harvest)

Example data:			Expected value is the sum of:			Standard deviation is the root of the sum of:			
	s	a	EV of i	harvest	EV		oak small	pine big	spruce big
oak small	6	0.3	31	16	$0.3 \cdot (31-16)$	oak small	$0.3 \cdot 0.3 \cdot 6 \cdot 6 \cdot (+1.00)$	$0.3 \cdot 0.2 \cdot 6 \cdot 18 \cdot (-0.07)$	$0.3 \cdot 0.5 \cdot 6 \cdot 19 \cdot (+0.05)$
pine big	18	0.2	103	9	$0.2 \cdot (103-9)$	pine big	$0.2 \cdot 0.3 \cdot 18 \cdot 6 \cdot (-0.07)$	$0.2 \cdot 0.2 \cdot 18 \cdot 18 \cdot (+1.00)$	$0.2 \cdot 0.5 \cdot 18 \cdot 19 \cdot (+0.80)$
spruce big	19	0.5	85	9	$0.5 \cdot (85-9)$	spruce big	$0.5 \cdot 0.3 \cdot 19 \cdot 6 \cdot (+0.05)$	$0.5 \cdot 0.2 \cdot 19 \cdot 18 \cdot (+0.80)$	$0.5 \cdot 0.5 \cdot 19 \cdot 19 \cdot (+1.00)$