Faustmann rotation and population dynamics in the presence of a risk of destructive events

Patrice LOISEL[1]

29-30th October 2009

[1] INRA UMR-ASB Montpellier, France

Introduction

- Management of natural resources : what is the optimal duration of cycle production ?
- Optimal production planning
- Optimal rotation : best sequence of harvesting

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Introduction

- Management of natural resources : what is the optimal duration of cycle production ?
- Optimal production planning
- Optimal rotation : best sequence of harvesting
- Presence of the Risk of a destructive event : Martell, Routledge (1980), Reed (1984 ...)

Faustmann rotation and Price uncertainty : Guttormsen (2001)

Faustmann Rotation

- Faustmann solution solves Rotation problems
- A new rotation started at the same time as the previous one ends.
- Impact of rotation to harvesting ?
- Faustmann solution based on dynamic models

$$\max_{h(.),T} J_0$$

Population Dynamics

Model of a representative individual :

Number $n(.)$: $\dfrac{dn(t)}{dt} = -(m(t) + h(t))n(t)$

Status $s(.)$: $\dfrac{ds(t)}{dt} = G(s(t), n(t))$

m : natural mortality

h : harvesting rate, $h(t) \leq \overline{h}$

Optimization problem (fixed term T) :

$$\begin{aligned}\max_{h(.)} G_0 &= [H(n(.), s(.), h(.), T) + V_0(n(T), s(T))]e^{-\delta T} \\ &= V(T)e^{-\delta T}\end{aligned}$$

Faustmann Rotation

Without risk :

$$J_0 = (J_0 + V(T) - c_1)e^{-\delta T}$$

Faustmann problem :

$$\max_T J_0 = \frac{V(T) - c_1}{e^{\delta T} - 1}$$

$$\max_{h(.),T} \frac{H(n(.), s(.), h(.), T) + V_0(n(T), s(T)) - c_1}{e^{\delta T} - 1}$$

In presence of risk

Epidemics occur in a Poisson process at rate λ (average rate per unit time)

Proportion salvageable x_t with $\alpha(t) = E(x_t)$
Three possibilities :

- For each event, stop and start a new cycle
  0 –> M(0,t_1) –> t_1 ($t_1 \leq T$)
- For each event, continue
  0 –> M(0,t_1) –> t_1 –> M(t_1,t_2) –> t_2 –> .. –> T
- Criteria $\begin{cases} \text{stop and start a new cycle} \\ \text{continue} \end{cases}$

In presence of risk : Case 1

$$J_0 = \int_0^T (J_0 + H(n(.), s(.), h(.), t) + \alpha(t) V_0(n(t), s(t)) - c_2) e^{-\delta t} dF(t)$$

$$+(J_0 + V_0(n(T), s(T)) - c_1) e^{-\delta T} (1 - F(T))$$

$$\max_{h(.),T} \frac{\delta + \lambda}{\delta} [\int_0^T [H(n(.), s(.), h(.), t) + \alpha(t) V_0(n(t), s(t))] \lambda e^{(\delta+\lambda)(T-t)} dt$$

$$+V_0(n(T), s(T)) - c_1]/(e^{(\delta+\lambda)T} - 1) - \frac{\lambda}{\delta} c_2$$

Optimal control

Using Pontryagin Maximum Principle :

$\theta(t) = R'(t) - (\lambda(1-\alpha(t)) + \delta + m(t))R(t) - c_n\lambda(1-\alpha(t))$

$\Theta(t) = \int_t^T e^{(\lambda+\delta)(T_*-u)-\overline{h}(u-t)}\theta(u)du, \qquad R(t) = p(s(t))$

Proposition

Assuming $\theta'(t) < 0$, let the optimal final term T_*, then :

- if $\theta(T_*) \geq 0$, $h_* \equiv 0$

- if $\theta(T_*) < 0$, it exists $0 \leq t_* < T_*$ such that : $h_*(t) = 0, t < t_*$ and $h_*(t) = \overline{h}, t > t_*$.

Moreover, if $\Theta(0) > 0$, t_* is given by :

$$\Theta(t_*) = \int_{t_*}^T e^{(\lambda+\delta)(T_*-u)-\overline{h}(u-t_*)}\theta(u)du = 0$$

else $t_* = 0$.

First conclusion

- $m(t)$ replaced by $m(t) + \lambda(1 - \alpha(t))$
- $\theta(t) - \theta_0(t) = -\lambda(1 - \alpha(t))(R(t) + c_n) < 0$
- Harvesting increases with risk in optimal management for a fixed final term T

And for respective optimal final term T ?

Simulation 1

$\lambda = 0.01, \delta = 0.03, \alpha = 0.5, \overline{h} = 0.05, R(t) = t, c_1 = 11.5$

Without risk :

$h = 0$ => $T_* = 32., J_0 = 12.72 n_0$

With the presence of a risk :

$h(t) = 0$ => $T_* = 30., J_0 = 4.98 n_0$
$h(t) > 0, 18.5 \leq t \leq 37.$ => $T_* = 37., J_0 = 5.12 n_0$

Higher final term T, the higher the probability of achieving the cycle low

Simulation 2

$\lambda = 0.02, \delta = 0.03, \alpha = 0.5, \overline{h} = 0.05, R'(t) = 0.9 + \dfrac{0.1 n_0}{n(t)}, c_1 = 11.5$

Without risk :

$h = 0$ => $T_* = 32.0, J_0 = 12.71 n_0$
$h(t) > 0, 27.5 \leq t \leq 32.5$ => $T_* = 32.5, J_0 = 12.72 n_0$

With the presence of a risk :

$h = 0$ => $T_* = 30.0, J_0 = 4.98 n_0$
$h(t) > 0, 27.5 \leq t \leq 31.5$ => $T_* = 31.5, J_0 = 5.00 n_0$
$h(t) > 0, 15. \leq t \leq 40.5$ => $T_* = 40.5, J_0 = 5.28 n_0$

In presence of risk : case 2

Same hypothesis. In case of event, the cycle continues.

$$\max_{h(.),T} \frac{H(\tilde{n}(.), s(.), h(.), T) + V_0(\tilde{n}(T), s(T)) - c_1}{e^{\delta T} - 1}$$

$$\frac{d\tilde{n}(t)}{dt} = -(m(t) + \lambda(1 - \alpha(t)) + h(t))\tilde{n}(t)$$

where : $\tilde{n}(t) = E_x(n(t))$

Conclusion

- Harvesting increases with risk
- Larger harvesting induces larger final term
- Final term can be larger with risk
- Density dependence
- Criterion of decision
- Influence of model complexity

THANK YOU FOR YOUR ATTENTION

Contact : Patrice LOISEL, patrice.loisel@supagro.inra.fr