Faustmann rotation and population dynamics in the presence of a risk of destructive events

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29-30th October 2009

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Introduction

- Management of natural resources: what is the optimal duration of cycle production?
- Optimal production planning
- Optimal rotation: best sequence of harvesting
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Faustmann rotation and Price uncertainty: Guttormsen (2001)
Faustmann Rotation

- Faustmann solution solves Rotation problems
- A new rotation started at the same time as the previous one ends.
- Impact of rotation to harvesting?
- Faustmann solution based on dynamic models

$$\max_{h(.), T} J_0$$
Population Dynamics

Model of a representative individual:

Number $n(.)$: \[
\frac{dn(t)}{dt} = -(m(t) + h(t))n(t)
\]

Status $s(.)$: \[
\frac{ds(t)}{dt} = G(s(t), n(t))
\]

$m$: natural mortality

$h$: harvesting rate, $h(t) \leq \bar{h}$

Optimization problem (fixed term $T$):

\[
\max_{h(\cdot)} G_0 = [H(n(.), s(.), h(.), T) + V_0(n(T), s(T))]e^{-\delta T}
\]

\[
= V(T)e^{-\delta T}
\]
Faustmann Rotation

Without risk:

\[ J_0 = (J_0 + V(T) - c_1)e^{-\delta T} \]

Faustmann problem:

\[
\max_T J_0 = \frac{V(T) - c_1}{e^{\delta T} - 1}
\]

\[
\max_{h(.), T} \frac{H(n(.), s(.), h(.), T) + V_0(n(T), s(T)) - c_1}{e^{\delta T} - 1}
\]
In presence of risk

Epidemics occur in a Poisson process at rate $\lambda$ (average rate per unit time)

Proportion salvageable $x_t$ with $\alpha(t) = E(x_t)$

Three possibilities:

- For each event, stop and start a new cycle
  $0 \rightarrow M(0, t_1) \rightarrow t_1$ ($t_1 \leq T$)

- For each event, continue
  $0 \rightarrow M(0, t_1) \rightarrow t_1 \rightarrow M(t_1, t_2) \rightarrow t_2 \rightarrow .. \rightarrow T$

- Criteria
  $\begin{cases} 
  \text{stop and start a new cycle} \\
  \text{continue}
  \end{cases}$
In presence of risk : Case 1

\[ J_0 = \int_0^T \left( J_0 + H(n(.), s(.), h(.), t) + \alpha(t) V_0(n(t), s(t)) - c_2 \right) e^{-\delta t} dF(t) \]

\[ + (J_0 + V_0(n(T), s(T)) - c_1)e^{-\delta T}(1 - F(T)) \]

\[ \max_{h(.), T} \frac{\delta + \lambda}{\delta} \left[ \int_0^T \left[ H(n(.), s(.), h(.), t) + \alpha(t) V_0(n(t), s(t)) \right] \lambda e^{(\delta + \lambda)(T - t)} dt \right] \]

\[ + V_0(n(T), s(T)) - c_1] / (e^{(\delta + \lambda)T} - 1) - \frac{\lambda}{\delta} c_2 \]
Optimal control

Using Pontryagin Maximum Principle:

\[ \theta(t) = R'(t) - (\lambda(1 - \alpha(t)) + \delta + m(t))R(t) - c_n\lambda(1 - \alpha(t)) \]

\[ \Theta(t) = \int_t^T e^{(\lambda + \delta)(T_*-u)-\overline{h}(u-t)}\theta(u)du, \quad R(t) = p(s(t)) \]

**Proposition**

Assuming \( \theta'(t) < 0 \), let the optimal final term \( T_* \), then:

- if \( \theta(T_*) \geq 0 \), \( h_* \equiv 0 \)
- if \( \theta(T_*) < 0 \), it exists \( 0 \leq t_* < T_* \) such that: \( h_*(t) = 0, t < t_* \) and \( h_*(t) = \overline{h}, t > t_* \).

Moreover, if \( \Theta(0) > 0 \), \( t_* \) is given by:

\[ \Theta(t_*) = \int_{t_*}^T e^{(\lambda + \delta)(T_*-u)-\overline{h}(u-t_*)}\theta(u)du = 0 \]

else \( t_* = 0 \).
First conclusion

- $m(t)$ replaced by $m(t) + \lambda(1 - \alpha(t))$
- $\theta(t) - \theta_0(t) = -\lambda(1 - \alpha(t))(R(t) + c_n) < 0$
- Harvesting increases with risk in optimal management for a fixed final term $T$

And for respective optimal final term $T$?
Simulation 1

\[ \lambda = 0.01, \delta = 0.03, \alpha = 0.5, \bar{h} = 0.05, R(t) = t, c_1 = 11.5 \]

Without risk:

\[ h = 0 \implies T_\ast = 32., J_0 = 12.72n_0 \]

With the presence of a risk:

\[ h(t) = 0 \implies T_\ast = 30., J_0 = 4.98n_0 \]
\[ h(t) > 0, 18.5 \leq t \leq 37. \implies T_\ast = 37., J_0 = 5.12n_0 \]

Higher final term \( T \), the higher the probability of achieving the cycle low.
Simulation 2

\[ \lambda = 0.02, \delta = 0.03, \alpha = 0.5, \bar{h} = 0.05, R'(t) = 0.9 + \frac{0.1n_0}{n(t)}, c_1 = 11.5 \]

Without risk:

\[ h = 0 \]
\[ h(t) > 0, 27.5 \leq t \leq 32.5 \] => \[ T_\ast = 32.0, J_0 = 12.71n_0 \]

With the presence of a risk:

\[ h = 0 \]
\[ h(t) > 0, 27.5 \leq t \leq 31.5 \] => \[ T_\ast = 31.5, J_0 = 5.00n_0 \]
\[ h(t) > 0, 15. \leq t \leq 40.5 \] => \[ T_\ast = 40.5, J_0 = 5.28n_0 \]
In presence of risk: case 2

Same hypothesis. In case of event, the cycle continues.

\[
\max_{h(\cdot), T} \frac{H(\tilde{n}(\cdot), s(\cdot), h(\cdot), T) + V_0(\tilde{n}(T), s(T)) - c_1}{e^{\delta T} - 1}
\]

\[
\frac{d\tilde{n}(t)}{dt} = -(m(t) + \lambda(1 - \alpha(t)) + h(t))\tilde{n}(t)
\]

where: \( \tilde{n}(t) = E_x(n(t)) \)
Conclusion

- Harvesting increases with risk
- Larger harvesting induces larger final term
- Final term can be larger with risk
- Density dependence
- Criterion of decision
- Influence of model complexity
THANK YOU FOR YOUR ATTENTION

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