OPTIMAL ROTATION WITH DECLINING DISCOUNT RATE

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Goods

Marginal utility (arbitrary) vs. Lapse of time (years)

Implicit discount rate

Marginal utility of consumption

Whole period discount rate

Instantaneous discount rate
Scenarios

- Slow growth scenario
- Fast growth scenario
- Mean of discount factors
- Discount factor based on mean discount rate
- Whole period discount rate
Perspectives

Lapse of time (years)

Discount factor

Discount rate

"Conservationists"  "Utilitarians"

Li and Löfgren's factor  Li and Löfgren's rate
Mentalities
Cumulative discount factor for 80 years:

\[
\frac{1}{1.035^{30}} \times \frac{1}{1.03^{45}} \times \frac{1}{1.025^5}
\]
Procedure:

Set rotations arbitrarily long;

shorten them iteratively until:

\[
\frac{\text{Revenue}_t}{(1 + r)^t} - \frac{\text{Revenue}_{t-1}}{(1 + r)^{t-1}} + \frac{\text{NPV}_{\text{next rotation}}}{(1 + r)^t} + \frac{\text{NPV}_{\text{next rotation}}}{(1 + r)^{t-1}} + \ldots
\]

\(< 0\)

Repeat for successor rotations until the answer stabilises (sometimes) ....
Rotation lengths (years) vs. Lapse of time (years)
How serious is it?
Reduced NPV from using the wrong criterion

<table>
<thead>
<tr>
<th>Criterion of profitability</th>
<th>NPV @ 1%</th>
<th>NPV @ 3.5%</th>
<th>NPV declining</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPV @ 1%</td>
<td>€2274</td>
<td>€1972</td>
<td>€2118</td>
</tr>
<tr>
<td>NPV @ 3.5%</td>
<td>€211</td>
<td>€264</td>
<td>€262</td>
</tr>
<tr>
<td>NPV declining</td>
<td>€303</td>
<td>€341</td>
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Dynamic inconsistency
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<tr>
<th>Event</th>
<th>Cash flow per hectare</th>
<th>Discounted value seen from time 2000</th>
<th>Discounted value seen from time 2080</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plant</td>
<td>£2000</td>
<td>£2000 × 1 = £2000</td>
<td></td>
</tr>
<tr>
<td>Fell at age 80</td>
<td>£6000</td>
<td>£6000 × 0.26613 = £1591</td>
<td>£6000 × 1 = £6000</td>
</tr>
<tr>
<td>Fell at age 120</td>
<td>£12000</td>
<td>£12 000 × 0.25215 = £3026</td>
<td>£12 000 × 0.35653 = £4278</td>
</tr>
</tbody>
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Effect of heavy initial discounting with high establishment cost
Dynamic consistency: shifting weight on objectives.
Naïve reassessment ...
... and smart reassessment
Reduced NPV from using the wrong criterion

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Stabilisation problems yet to be resolved:

- dealing with stepped discount functions
- incorporating large formation costs
- adding an infinite series of rotations beyond 1000 years.

Not problems of constructing formulas, but ...
... of structuring calculation sequence.
Do the one from DXX3608 with multiple factors. Try in KulaLiLof in Excel|Livepaps|Discount? Just do a factor which is \( \frac{(Kula+LiLof)/2+1}{2} \). Should be quick enough: copy graph and put on a further factor and discount rate.

How about the minimum compensation vs maximum endowment thing?