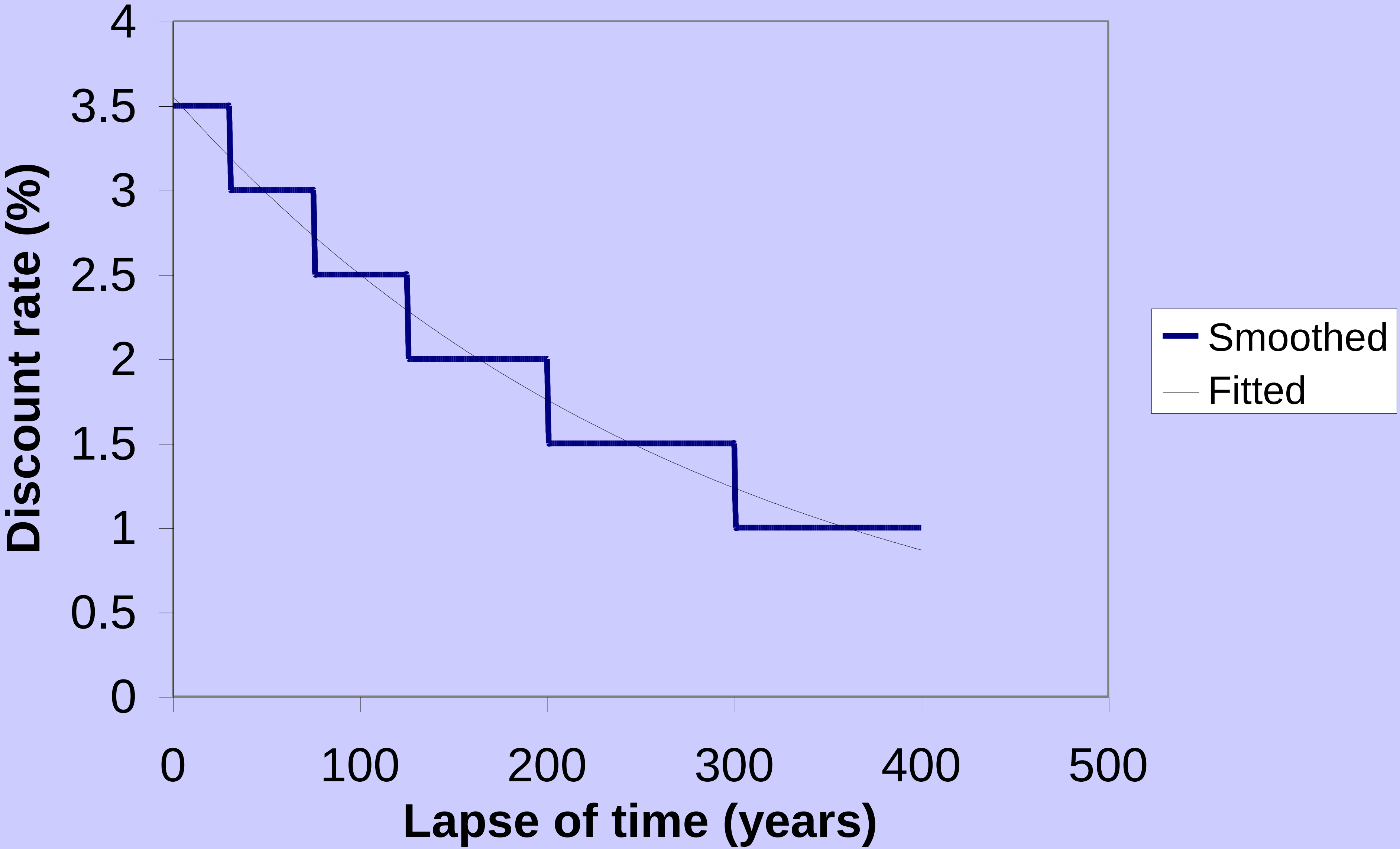
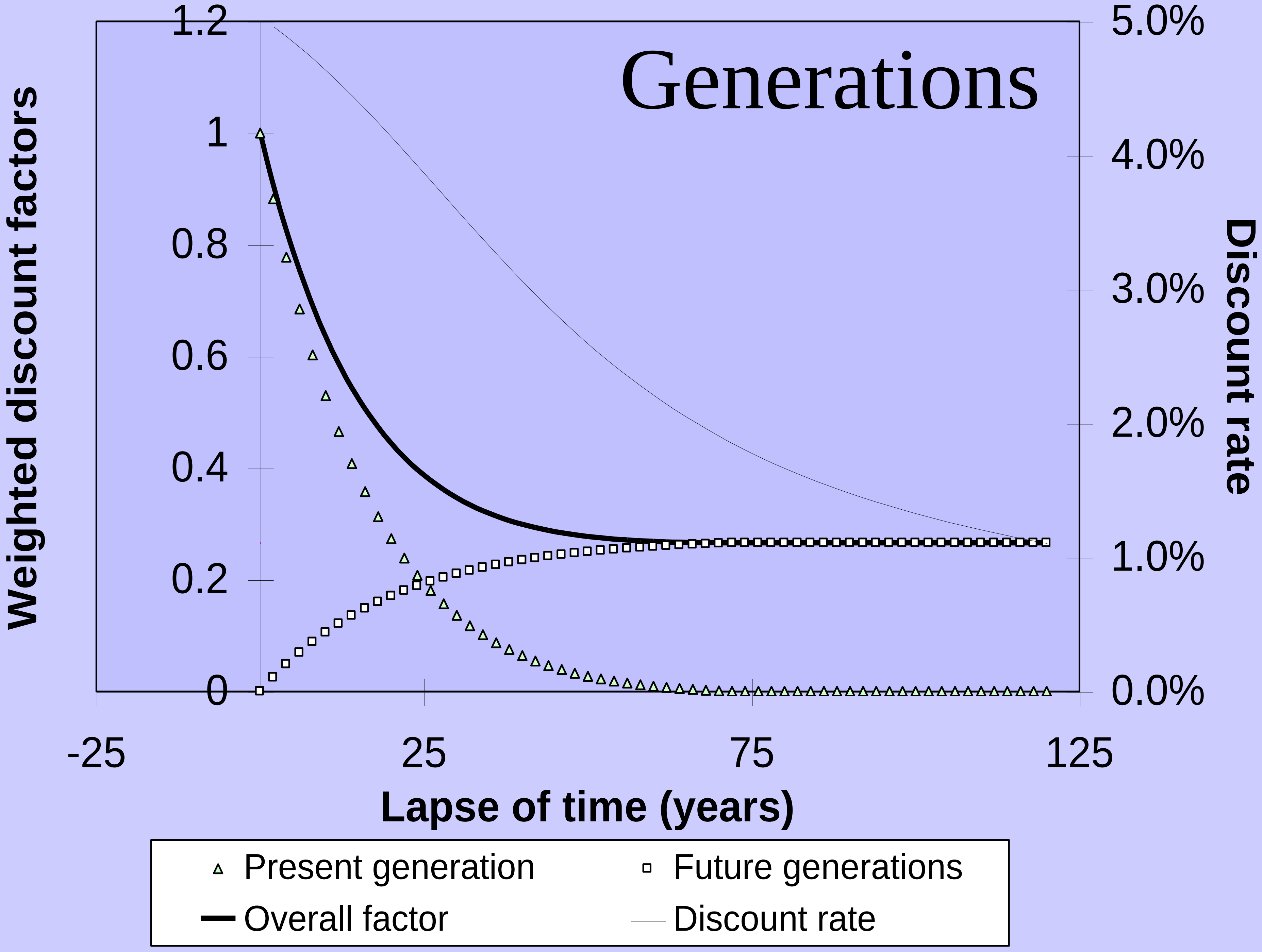
OPTIMAL ROTATION WITH DECLINING DISCOUNT RATE

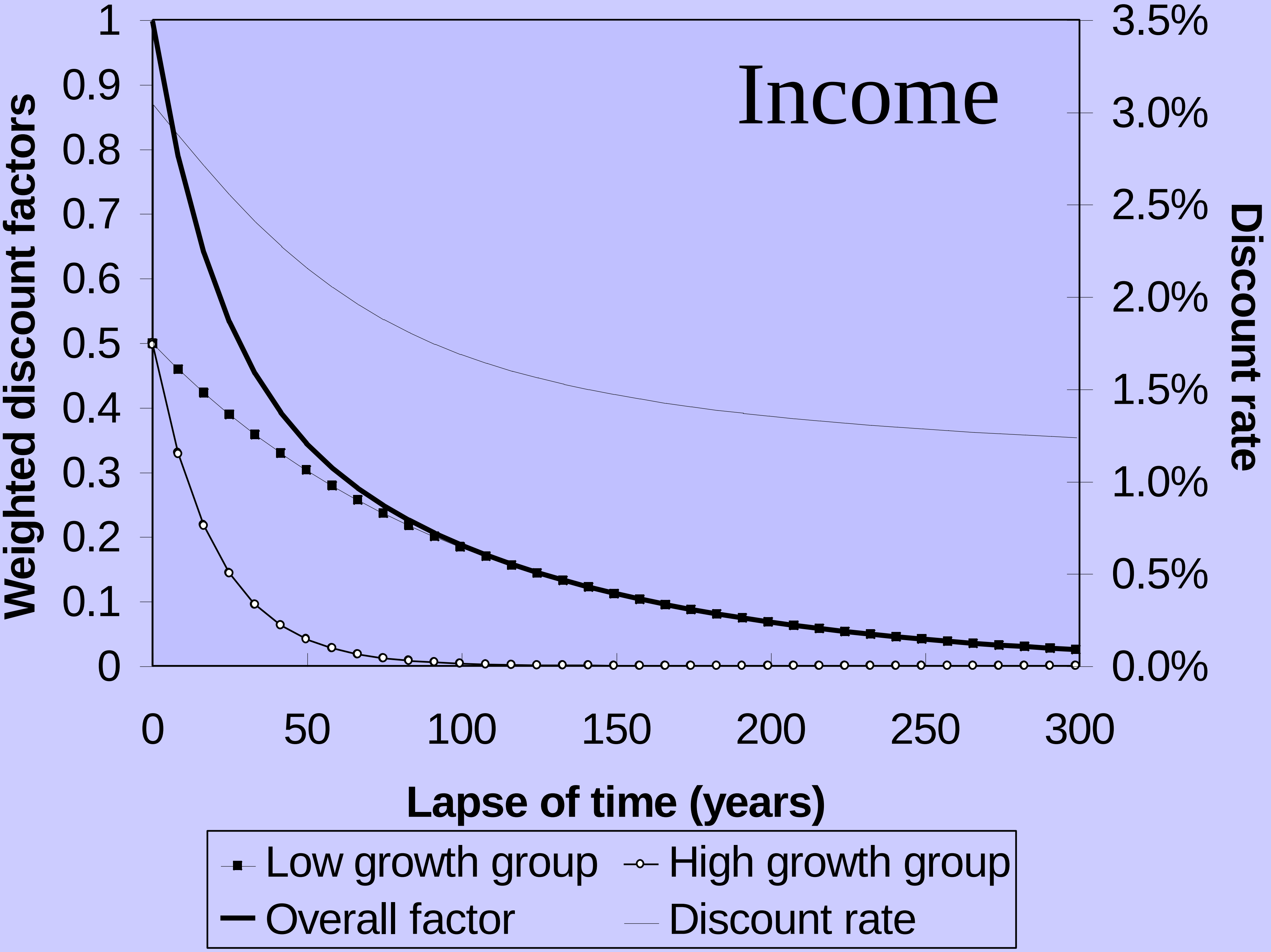
Colin Price
School of the Environment, Natural Resources and Geography
Bangor University
United Kingdom

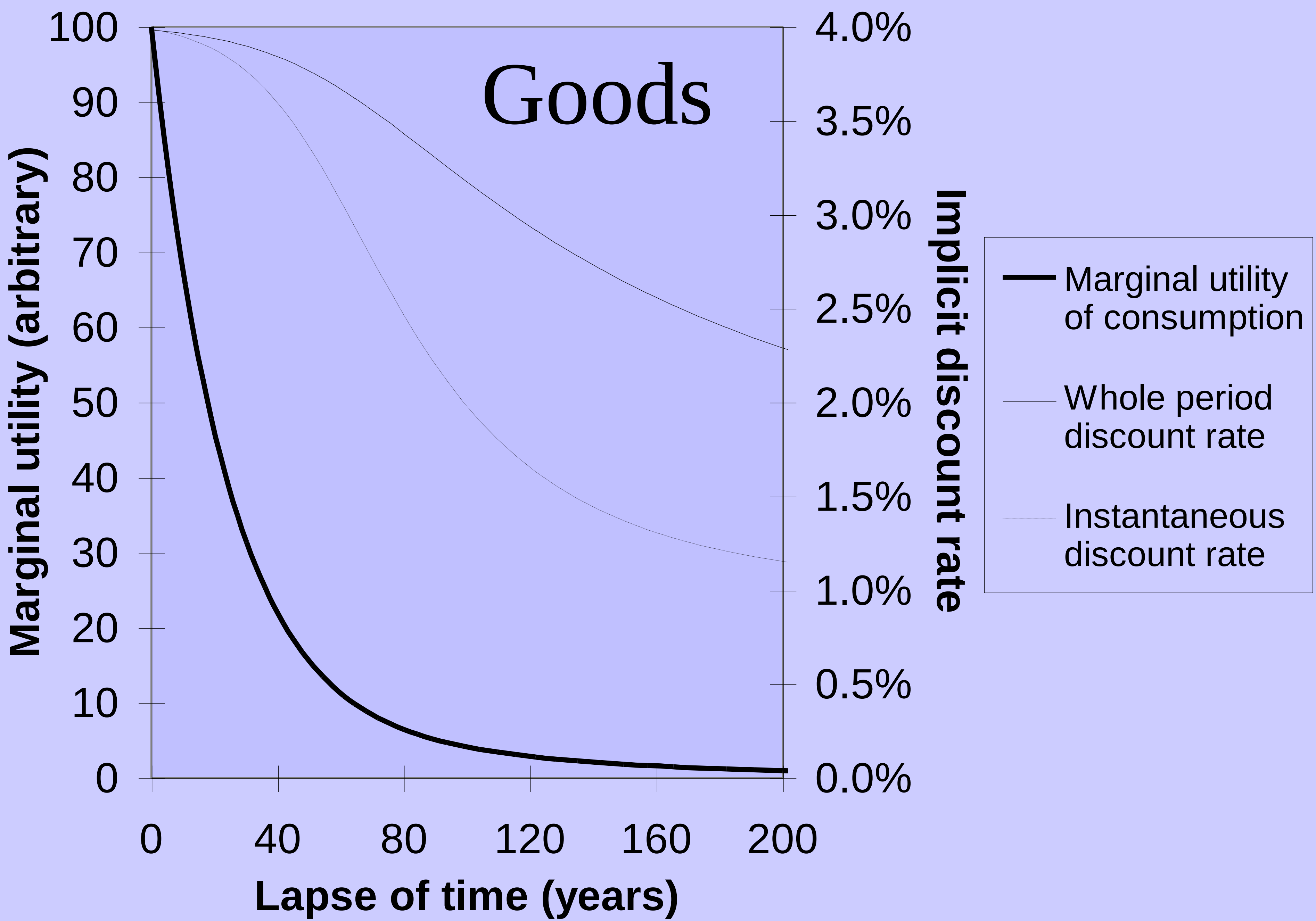


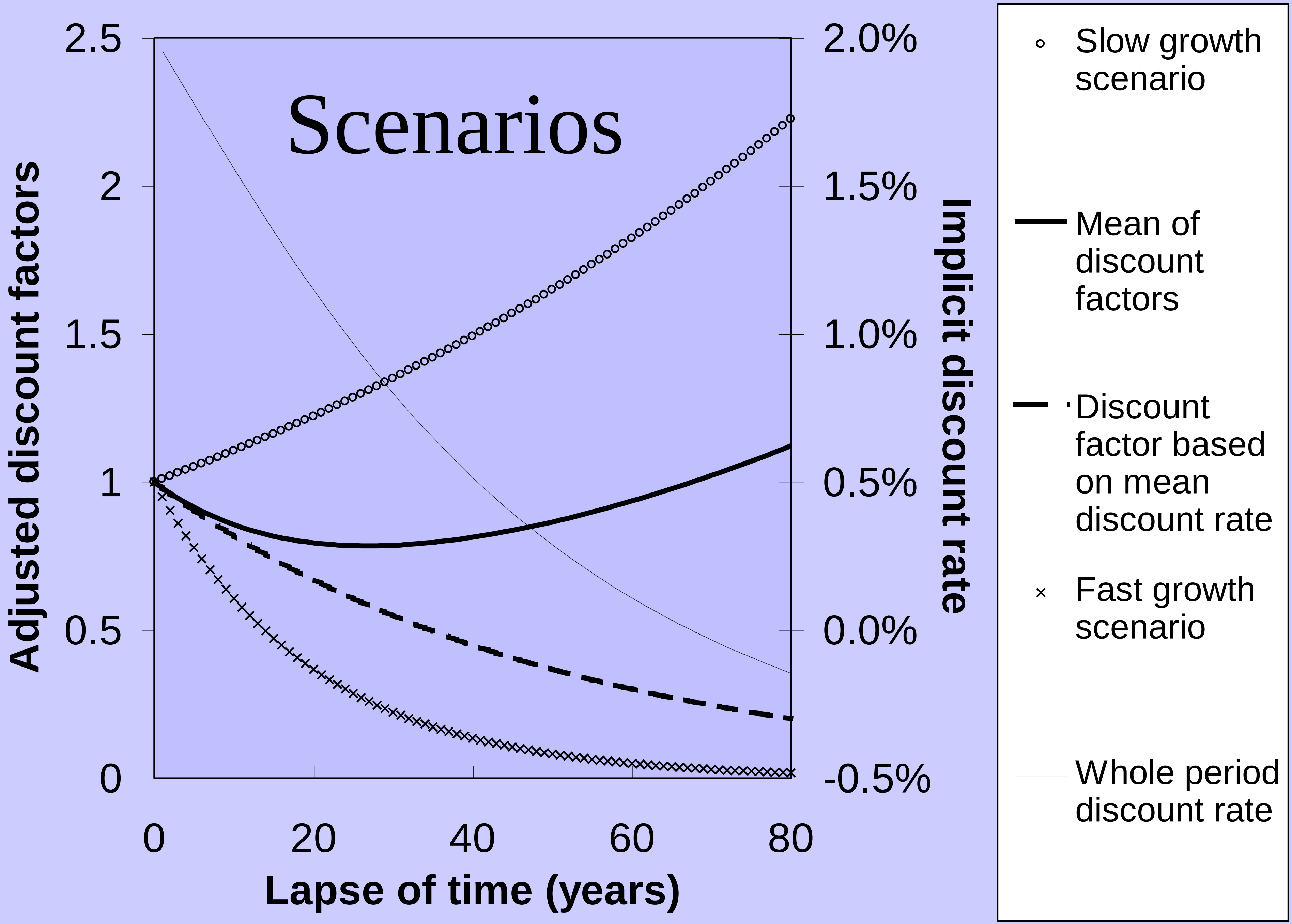
4
3.5
3
2.5
2
1.5
1
0.5
0
Discount rate (%)
0
100
200
300
400
500
Lapse of time (years)
Smoothed
Fitted



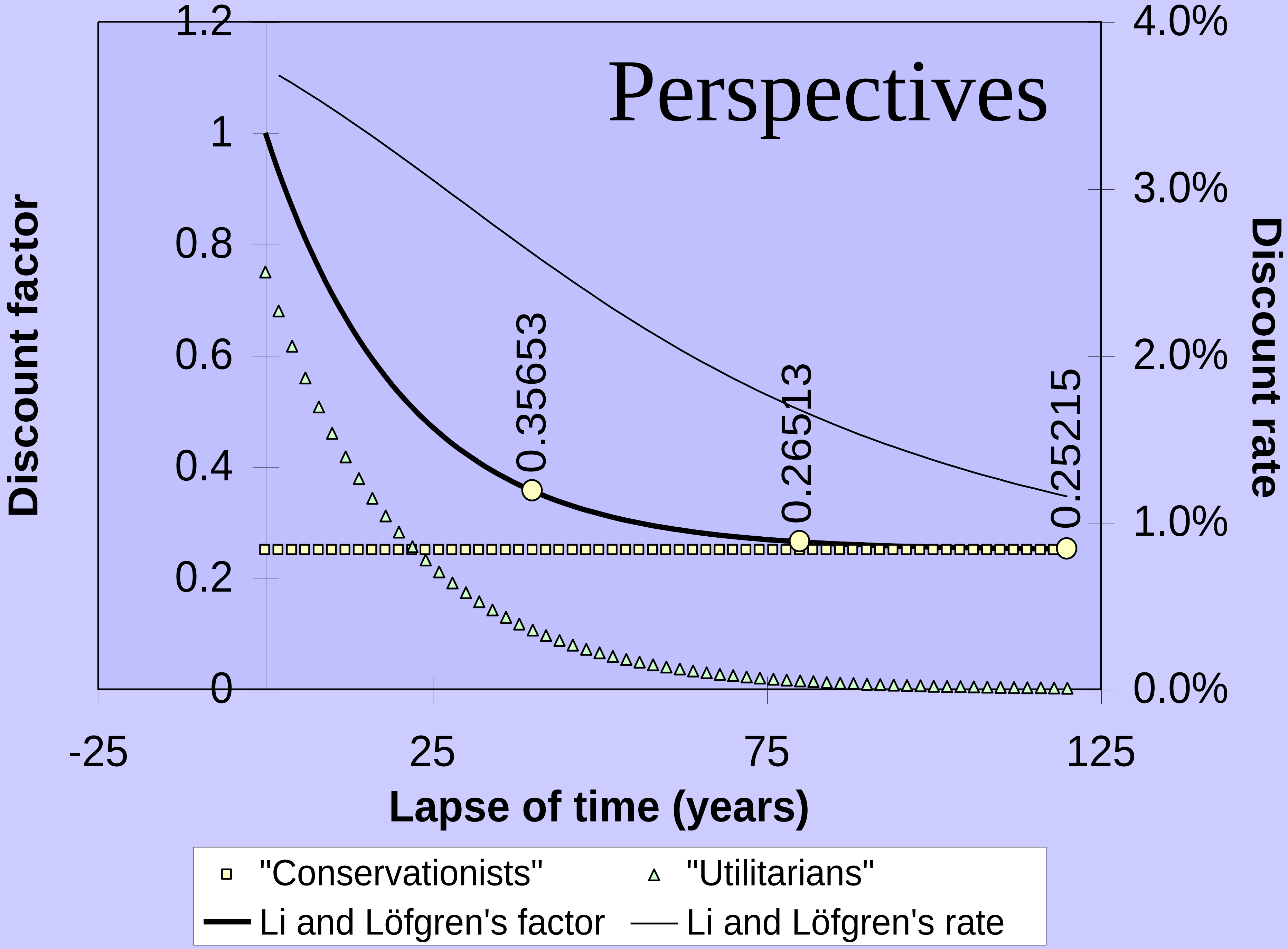
Generations
Weighted discount factors
Discount rate
Lapse of time (years)
1.2
1
0.8
0.6
0.4
0.2
0
5.0%
4.0%
3.0%
2.0%
1.0%
0.0%
-25
25
75
125
Present generation
Future generations
Overall factor
Discount rate

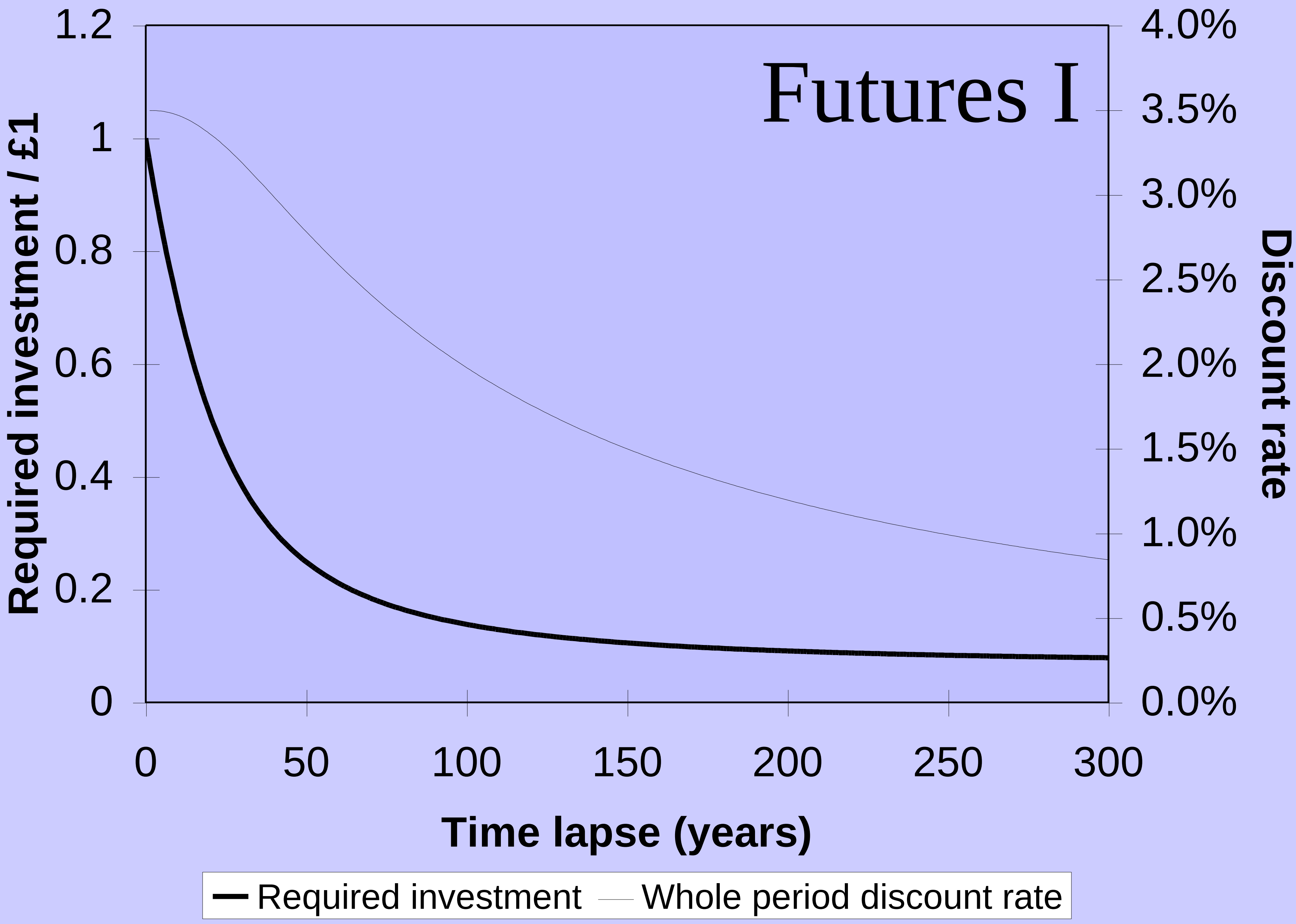

Income
Weighted discount factors
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Discount rate
3.5%
3.0%
2.5%
2.0%
1.5%
1.0%
0.5%
0.0%
0
50
100
150
200
250
300
Lapse of time (years)
Low growth group
High growth group
Overall factor
Discount rate


Goods
Marginal utility (arbitrary)
100
90
80
70
60
50
40
30
20
10
0
4.0%
3.5%
3.0%
2.5%
2.0%
1.5%
1.0%
0.5%
0.0%
Implicit discount rate
0
40
80
120
160
200
Lapse of time (years)
Marginal utility of consumption
Whole period discount rate
Instantaneous discount rate

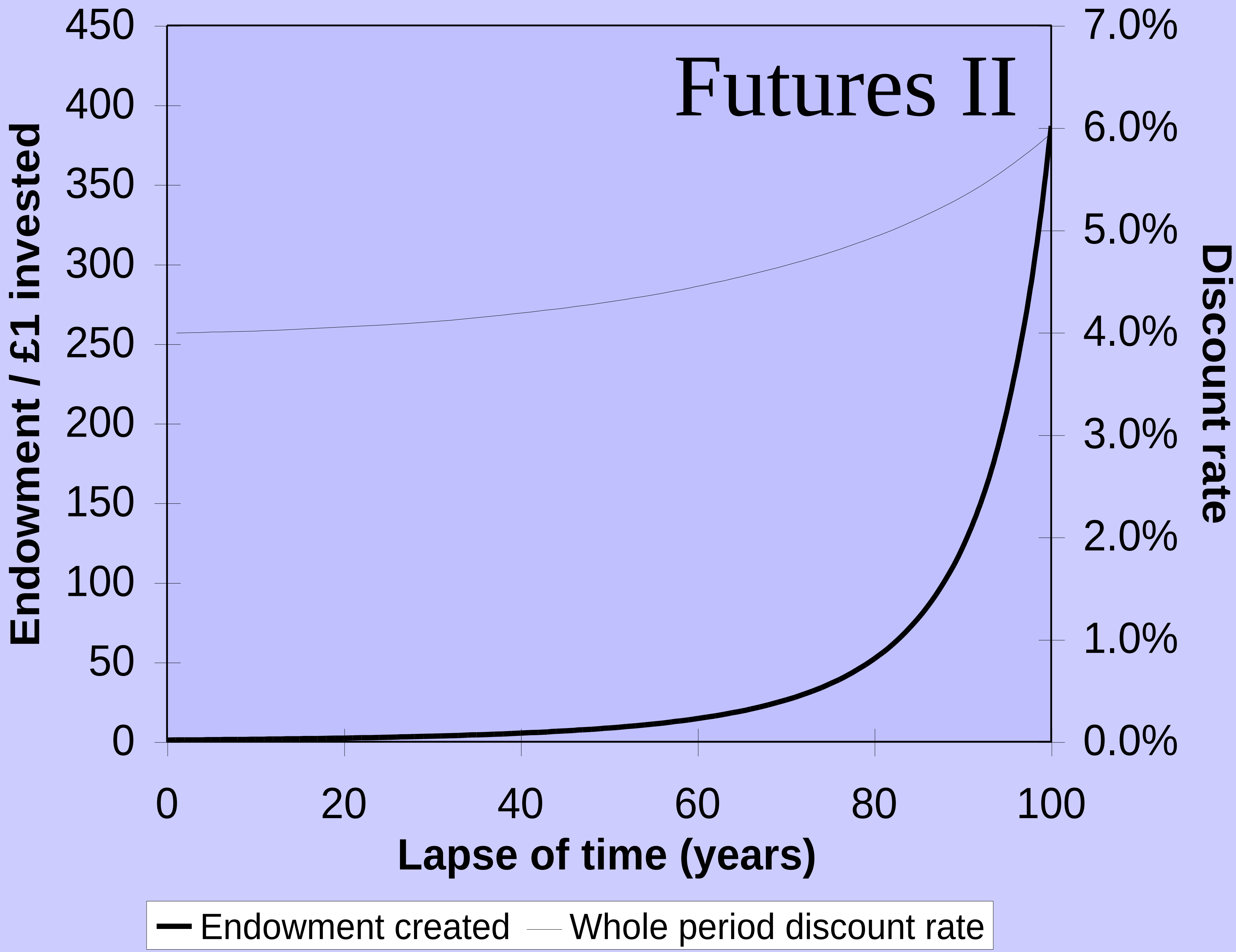


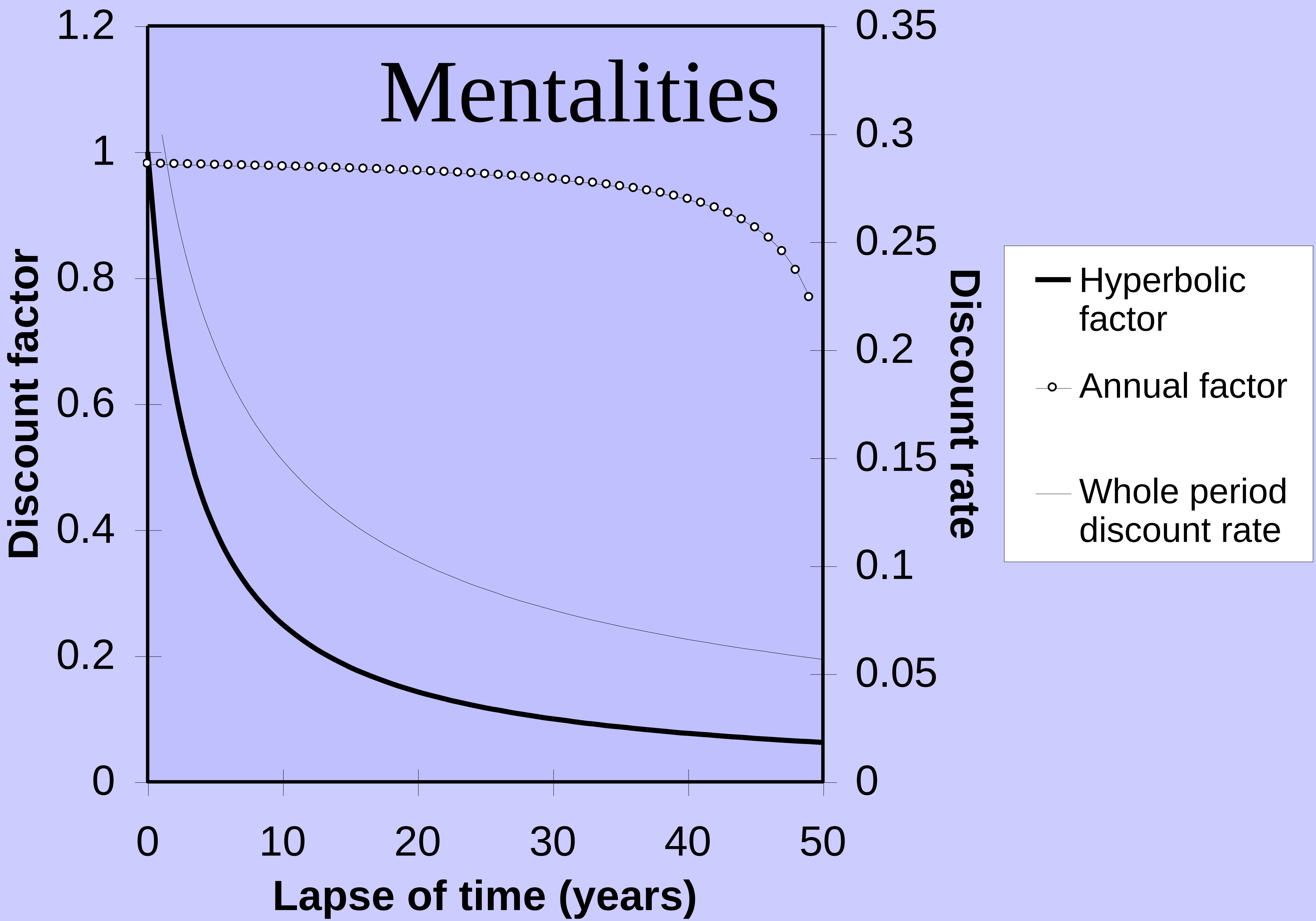
Scenarios
Adjusted discount factors
Implicit discount rate
Lapse of time (years)
2.5
2
1.5
1
0.5
0
2.0%
1.5%
1.0%
0.5%
0.0%
-0.5%
0
20
40
60
80
Slow growth scenario
Mean of discount factors
Discount factor based on mean discount rate
Fast growth scenario
Whole period discount rate


Perspectives
Discount factor
Discount rate
Lapse of time (years)
1.2
1
0.8
0.6
0.4
0.2
0
4.0%
3.0%
2.0%
1.0%
0.0%
-25
25
75
125
0.35653
0.26513
0.25215
"Conservationists"
"Utilitarians"
Li and Löfgren's factor
Li and Löfgren's rate

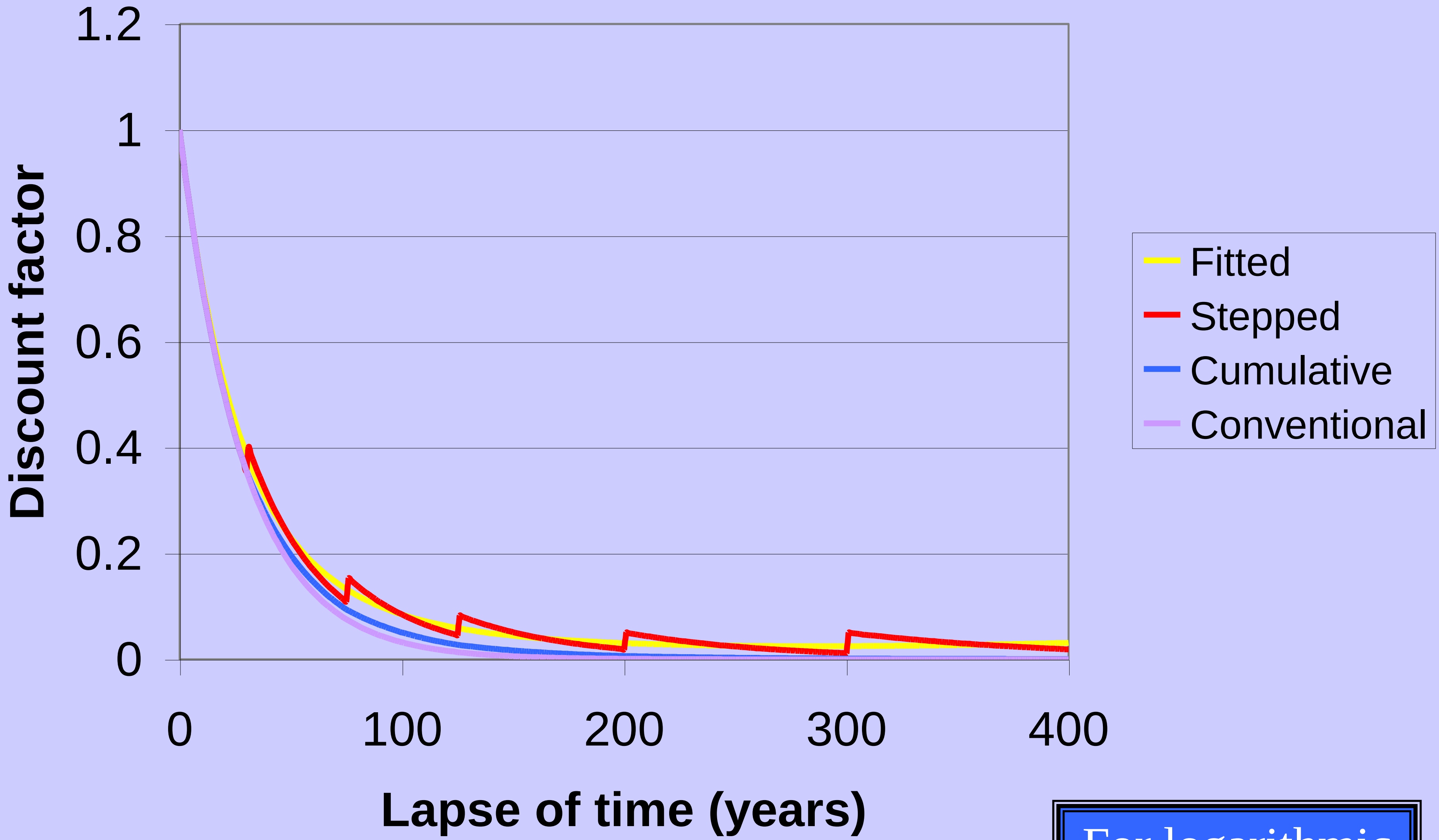


Futures I
Required investment / £1
Discount rate
Time lapse (years)
1.2
1
0.8
0.6
0.4
0.2
0
4.0%
3.5%
3.0%
2.5%
2.0%
1.5%
1.0%
0.5%
0.0%
0
50
100
150
200
250
300
Required investment
Whole period discount rate

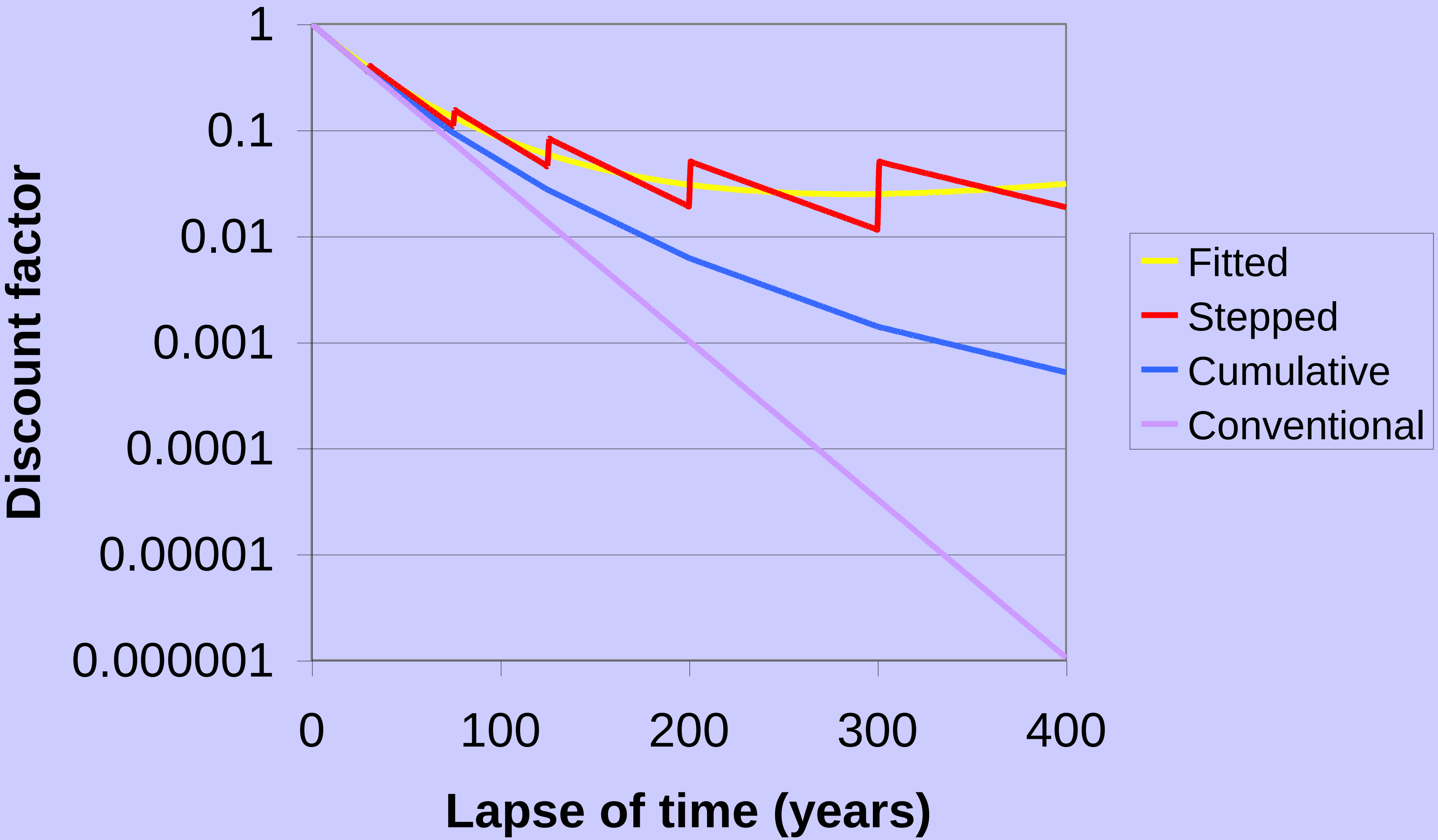

Futures II
Endowment / £1 invested
Discount rate
Lapse of time (years)
0
50
100
150
200
250
300
350
400
450
0.0%
1.0%
2.0%
3.0%
4.0%
5.0%
6.0%
7.0%
0
20
40
60
80
100
Endowment created
Whole period discount rate



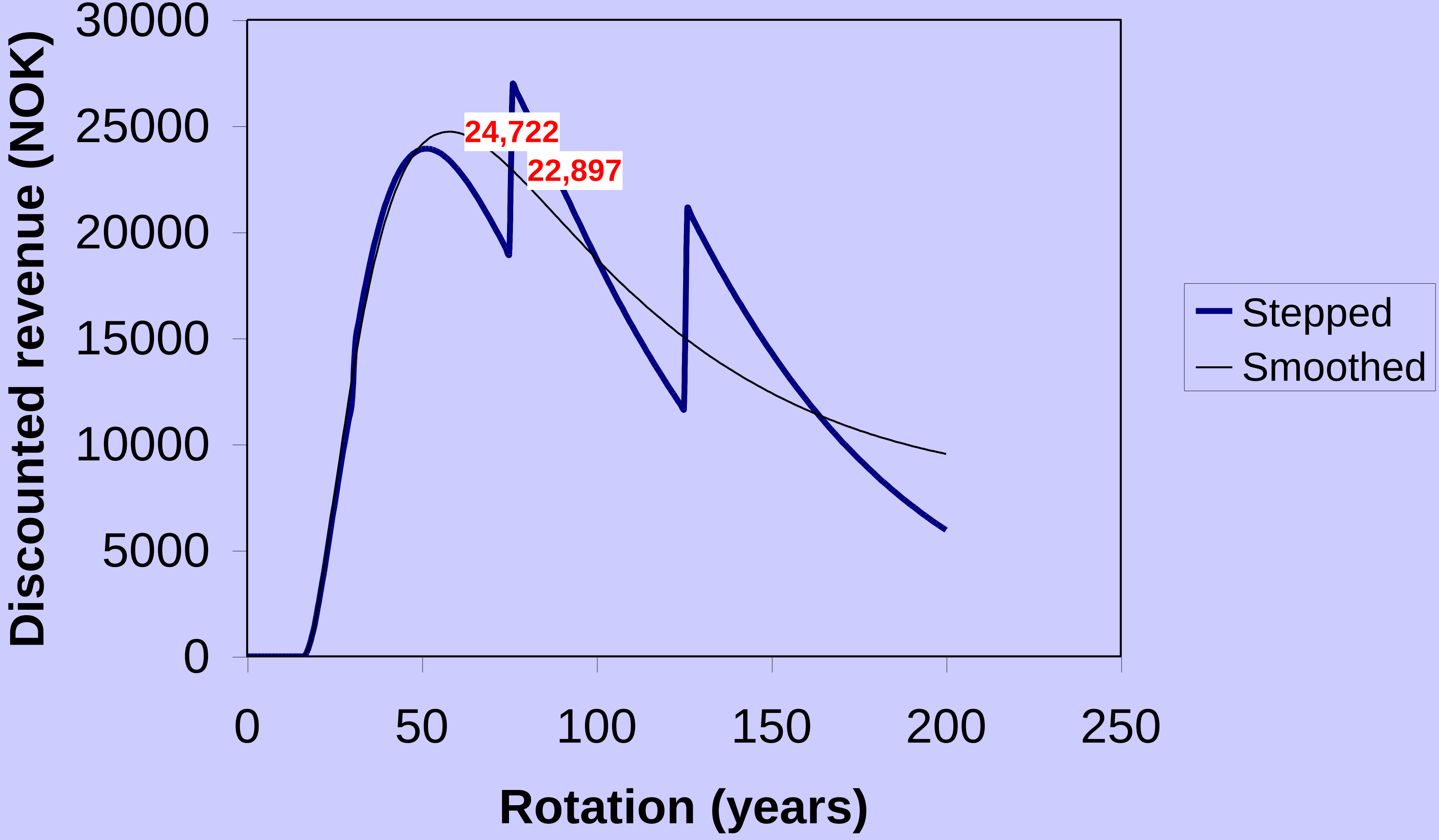
Mentalities
1.2
1
0.8
0.6
0.4
0.2
0
Discount factor
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Discount rate
0
10
20
30
40
50
Lapse of time (years)
Hyperbolic factor
Annual factor
Whole period discount rate



For logarithmic scale click here

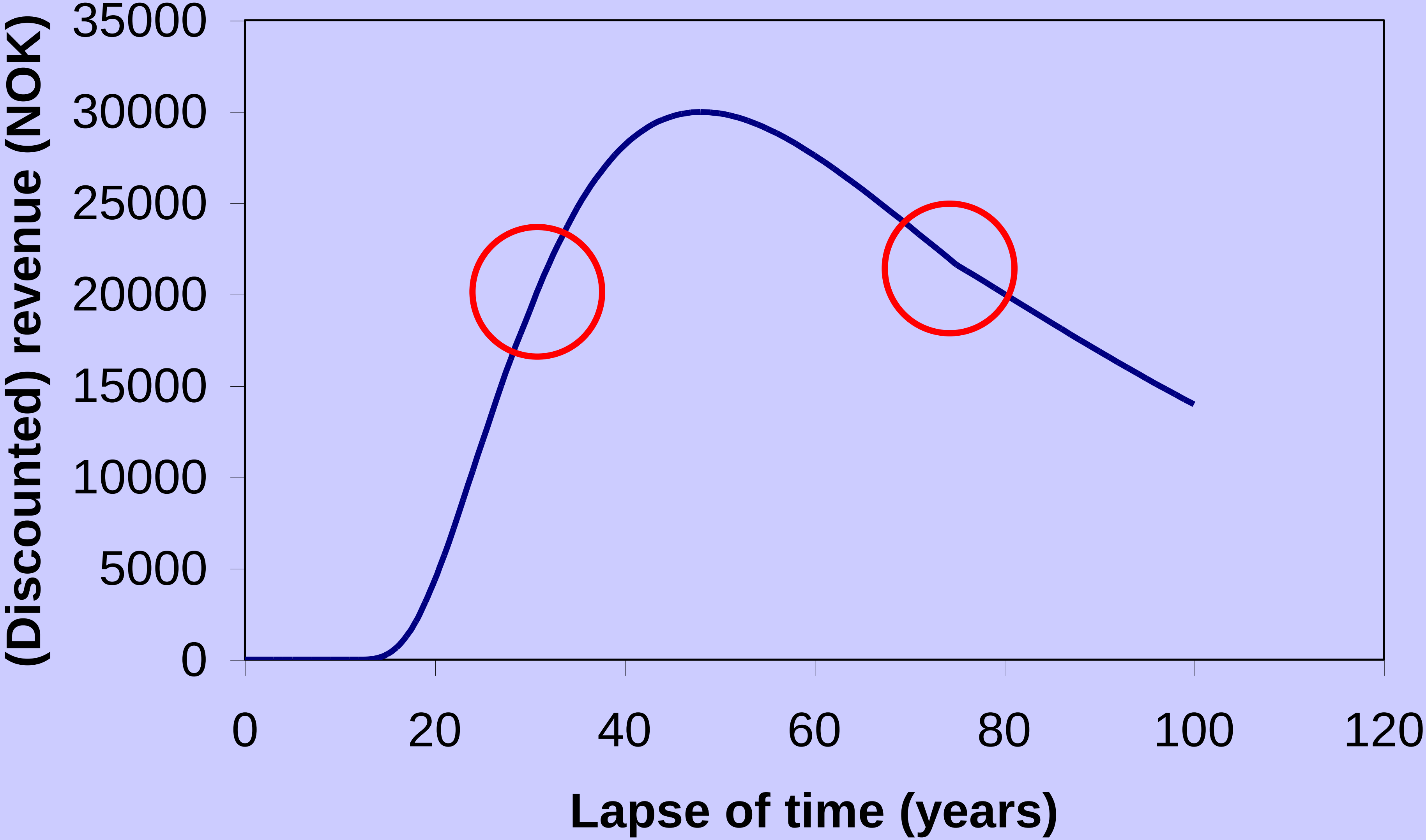


1
0.1
0.01
0.001
0.0001
0.00001
0.000001
Discount factor
0
100
200
300
400
Lapse of time (years)
Fitted
Stepped
Cumulative
Conventional


Discounted revenue (NOK)
30000
25000
20000
15000
10000
5000
0
0
50
100
150
200
250
Rotation (years)
24,722
22,897
Stepped
Smoothed

Cumulative discount factor for 80 years:

$$\frac{1}{1.035^{30}} \times \frac{1}{1.03^{45}} \times \frac{1}{1.025^{5}}$$


(Discounted) revenue (NOK)
35000
30000
25000
20000
15000
10000
5000
0
0
20
40
60
80
100
120
Lapse of time (years)

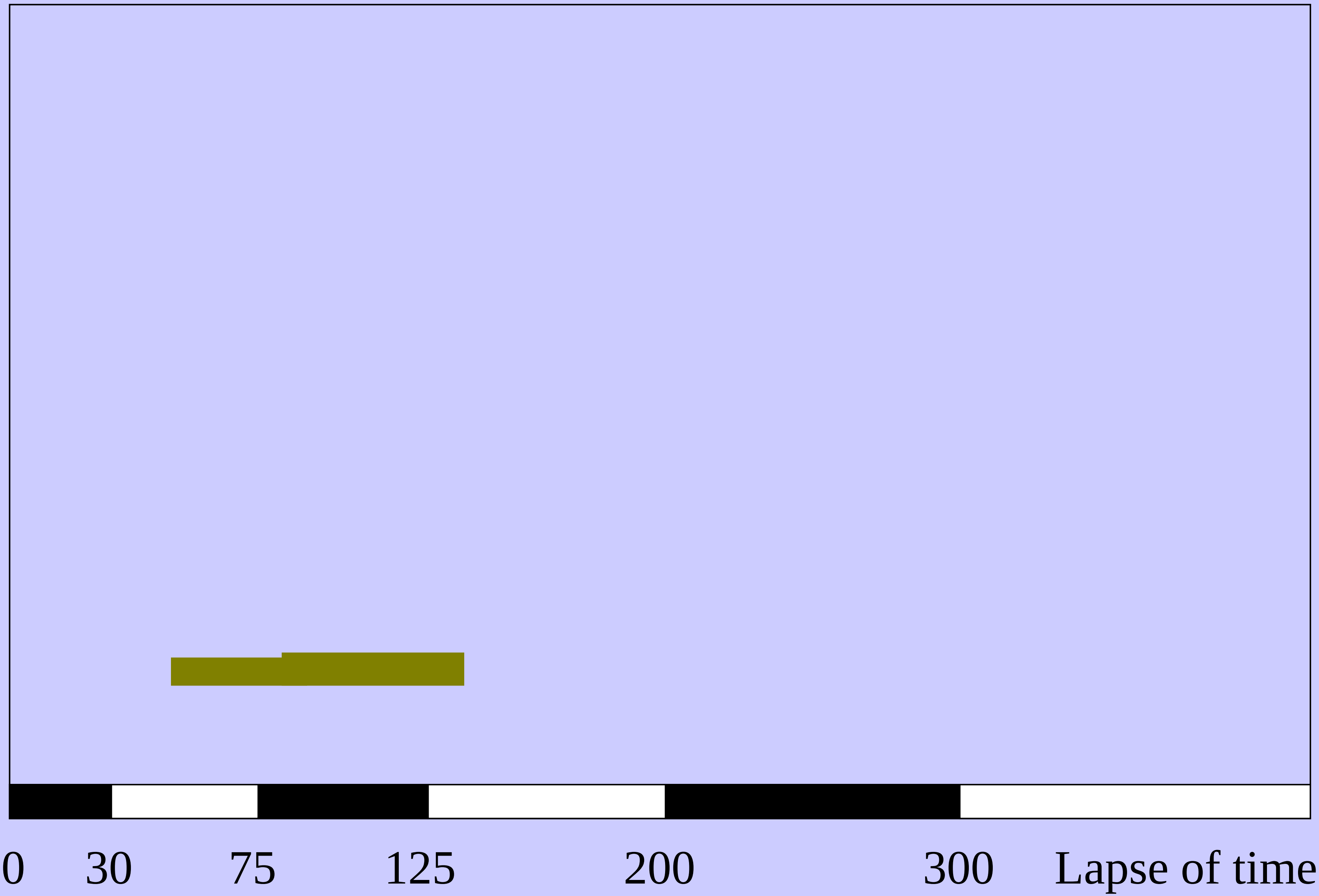
Revenue
at age *T*

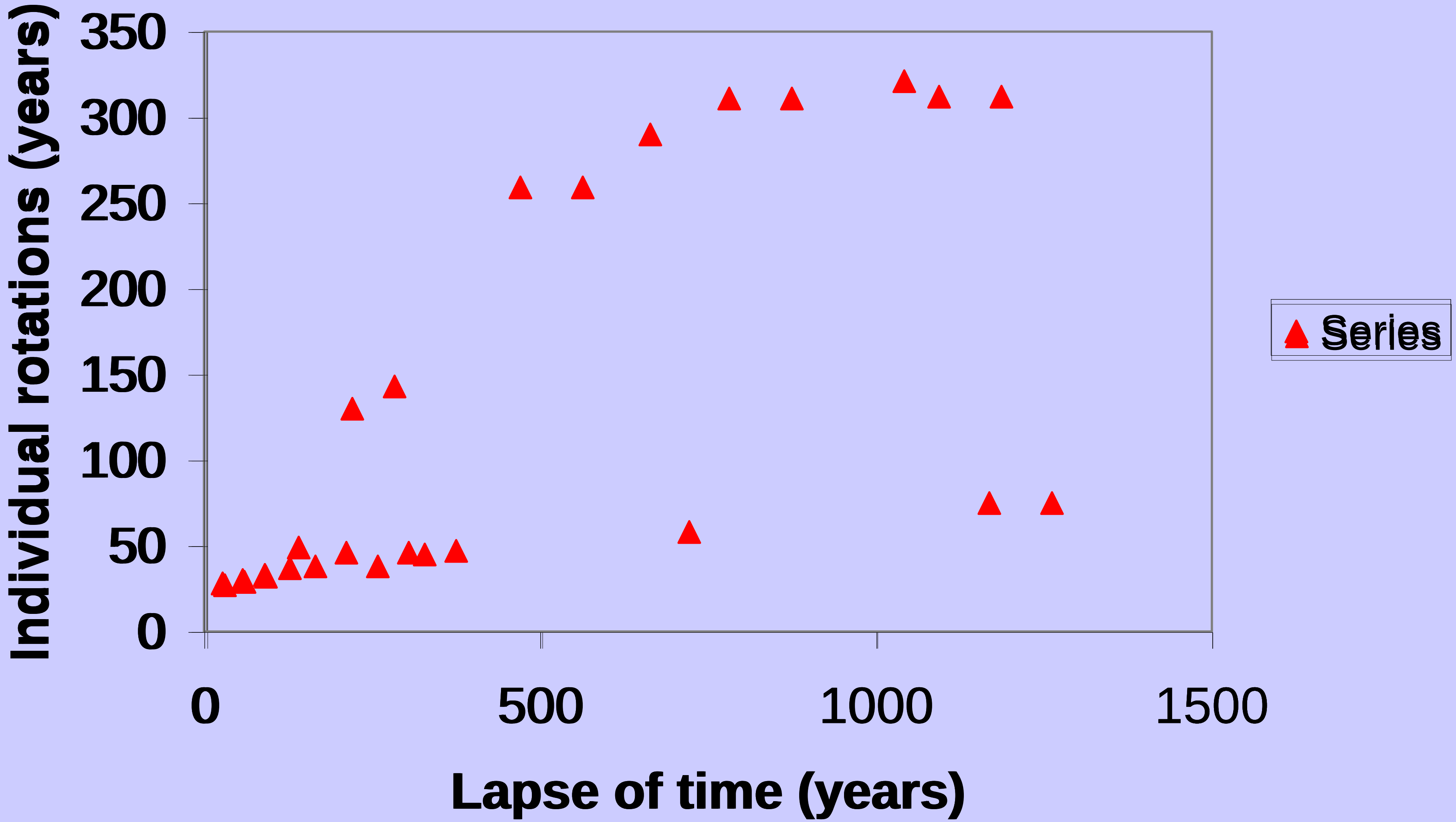
Revenue
at age *T*–1

NPV of
rotation II

NPV of
rotation III

NPV of
rotation IV


0
30
75
125
200
300
Lapse of time



Individual rotations (years)
350
300
250
200
150
100
50
0
0
500
1000
1500
Lapse of time (years)
Series

Procedure:

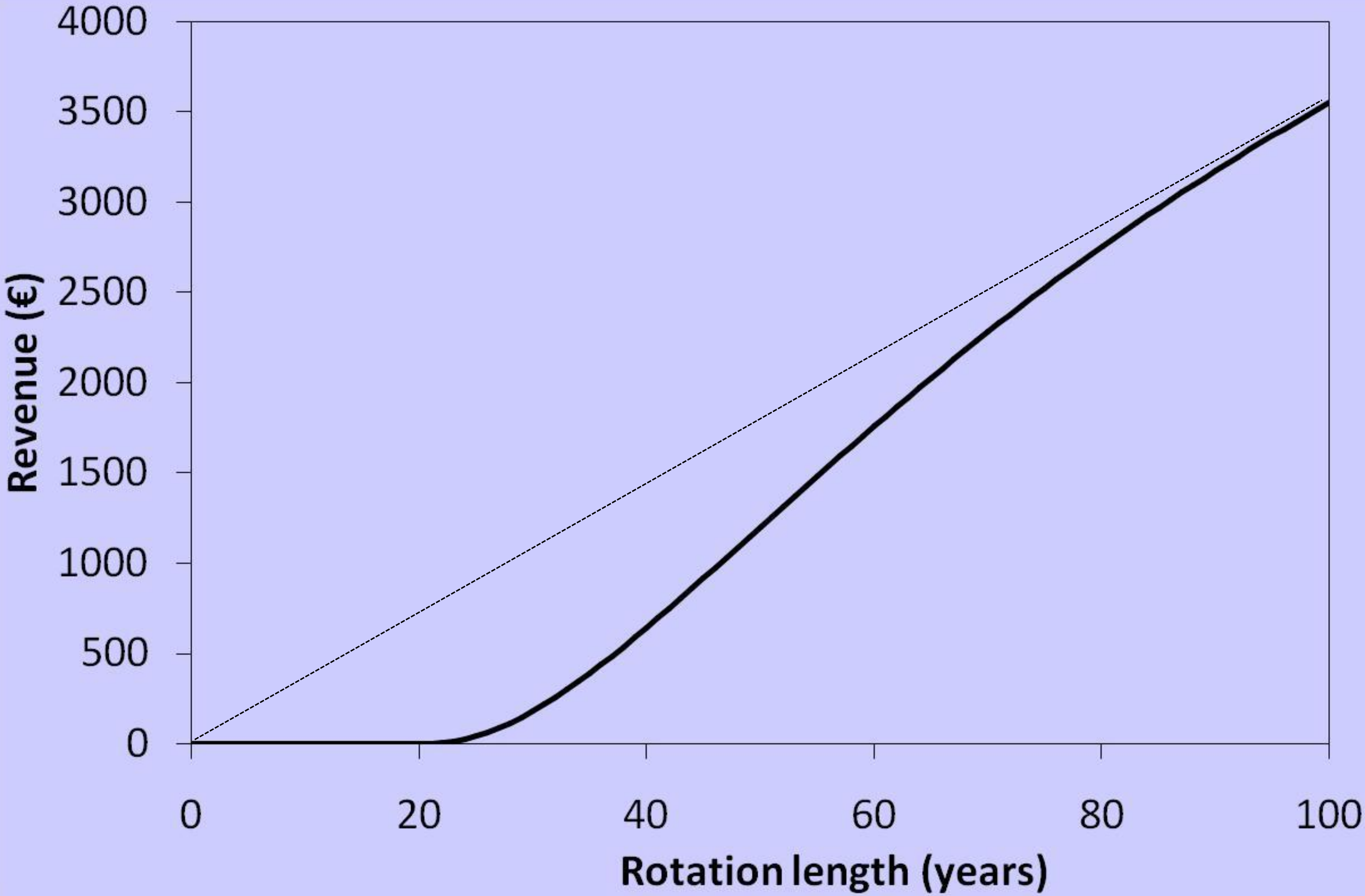
Set rotations arbitrarily long;

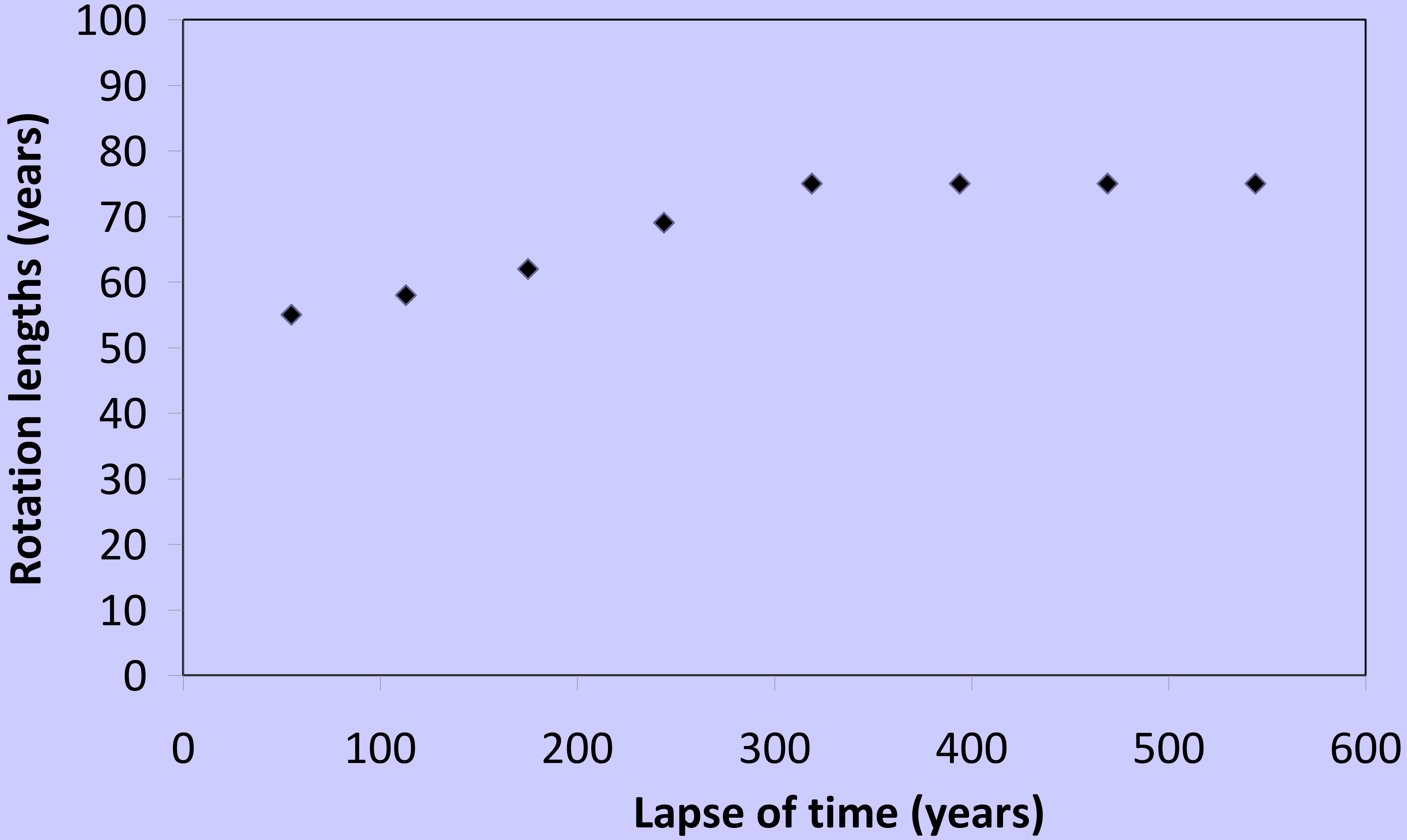
shorten them iteratively until:

$$\frac{[\text{Revenue}_{t}]}{(1+r)^{t}} - \frac{[\text{Revenue}_{t-1}]}{(1+r)^{(t-1)}} + \frac{[\text{NPV}_{\text{next rotation}}]}{(1+r)^{t}} + \frac{[\text{NPV}_{\text{next rotation}}]}{(1+r)^{(t-1)}} + \dots$$

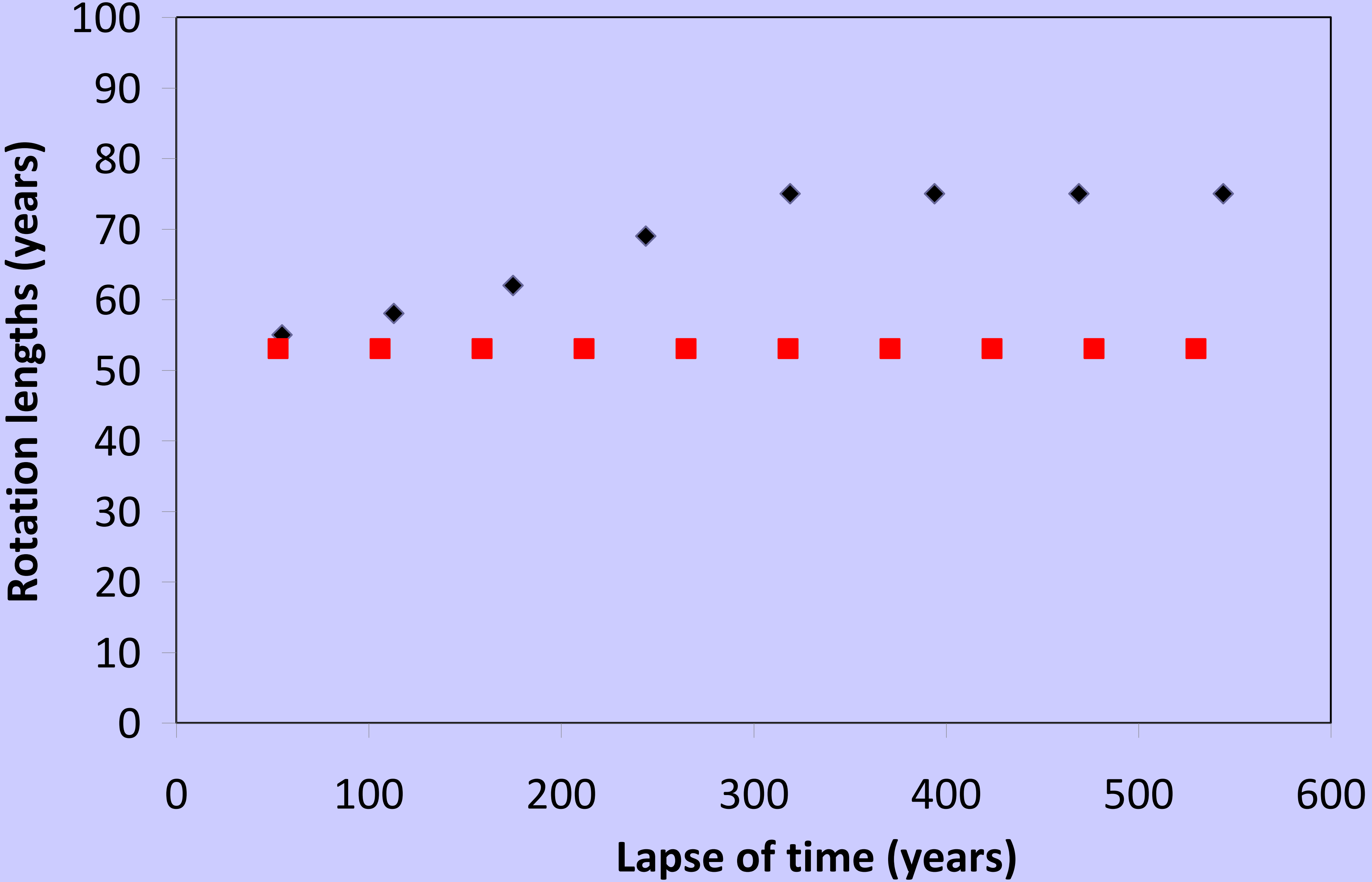
$$< 0$$

Repeat for successor rotations until the answer stabilises (sometimes) ....

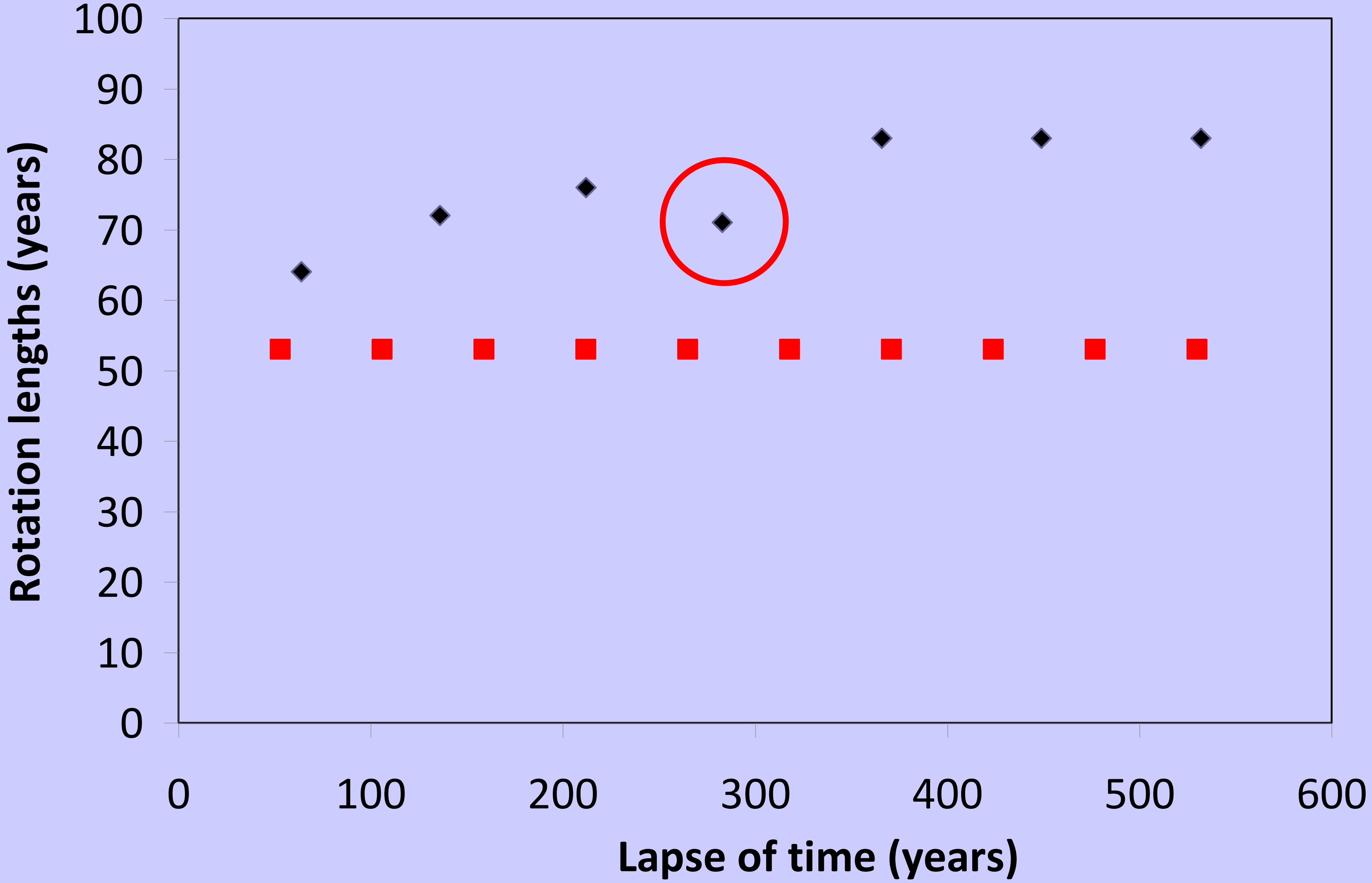

4000
3500
3000
2500
2000
1500
1000
500
0
Revenue (€)
0
20
40
60
80
100
Rotation length (years)



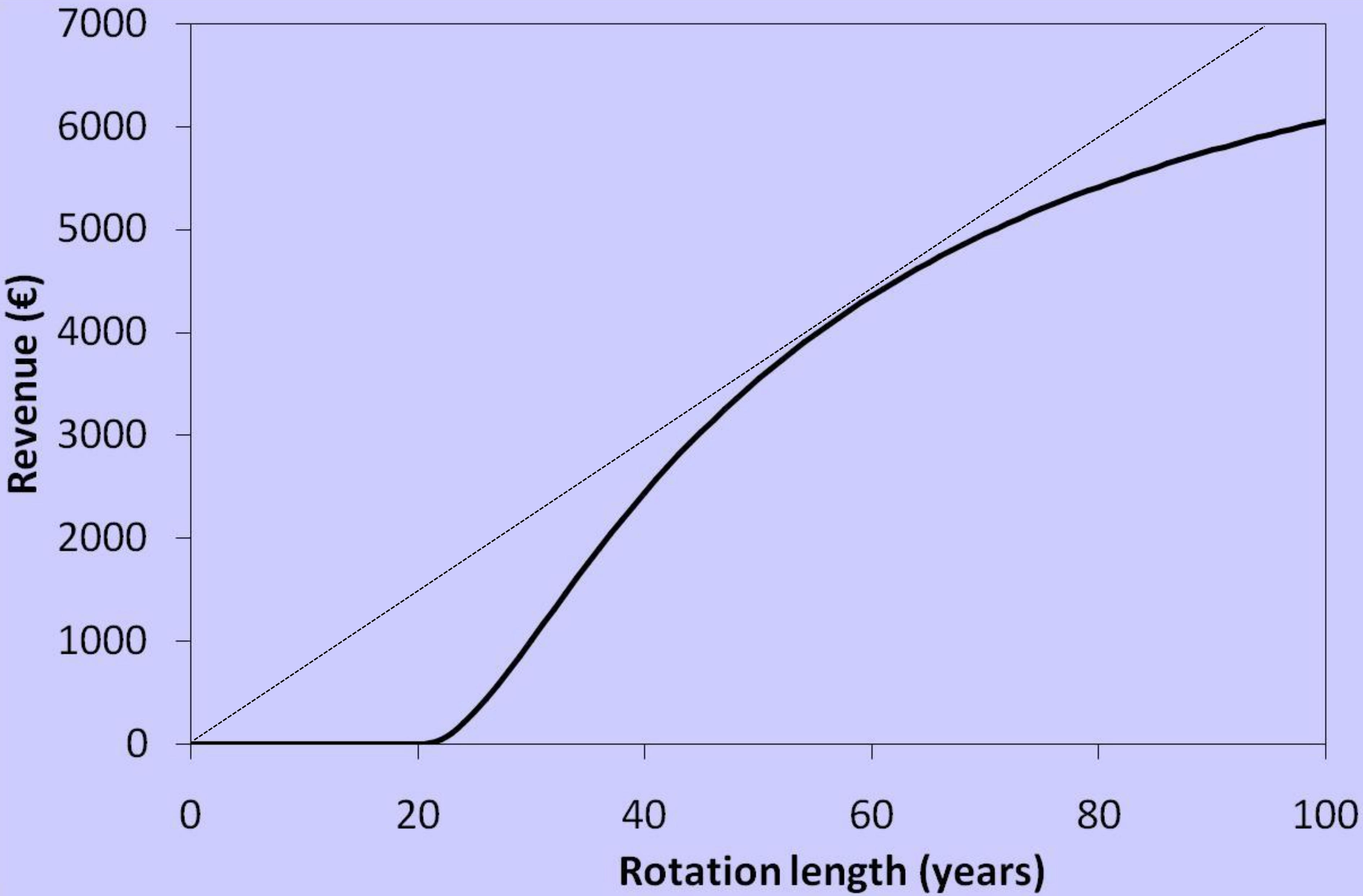
Rotation lengths (years)
100
90
80
70
60
50
40
30
20
10
0
0
100
200
300
400
500
600
Lapse of time (years)

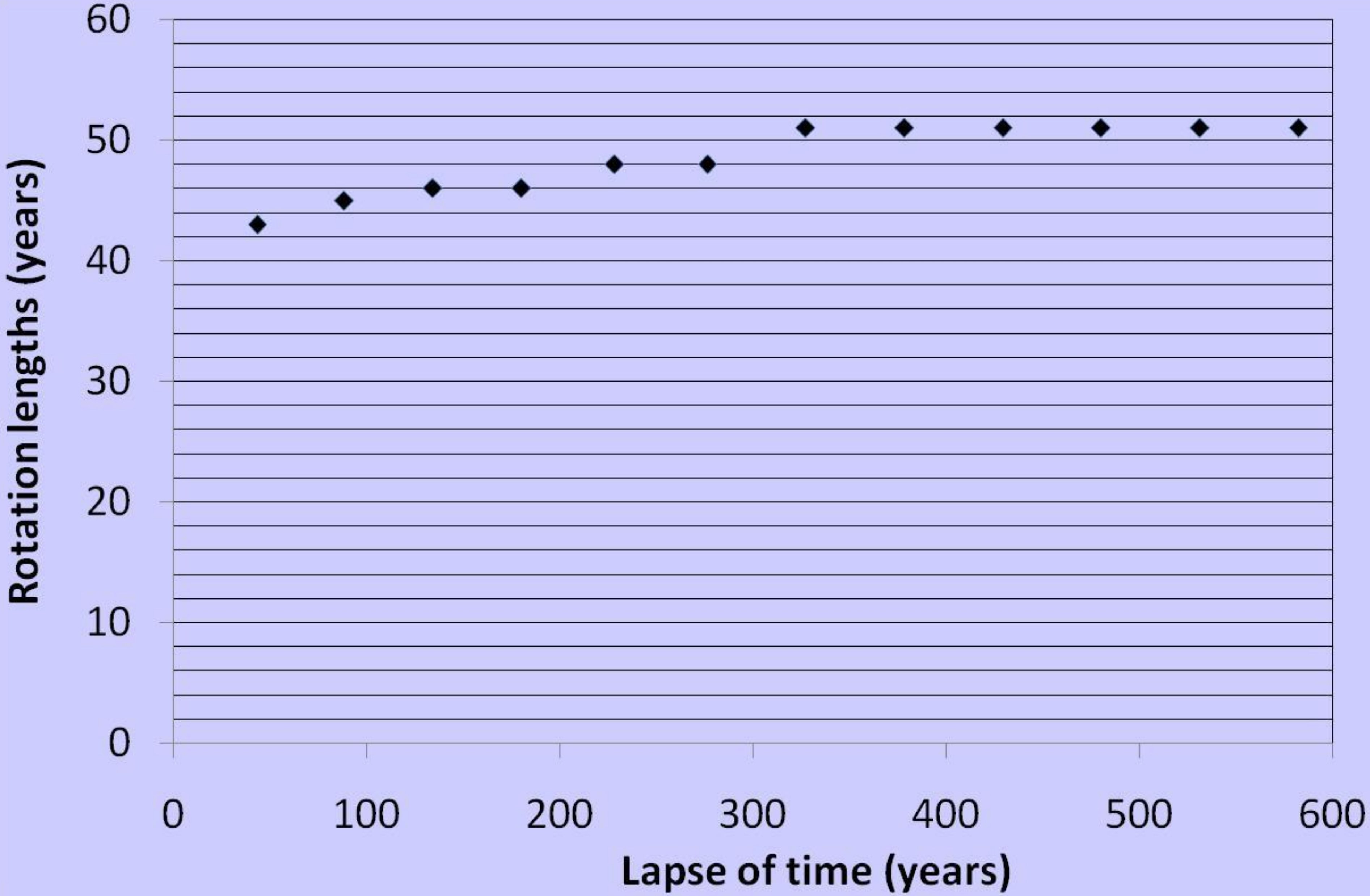


Rotation lengths (years)
100
90
80
70
60
50
40
30
20
10
0
0
100
200
300
400
500
600
Lapse of time (years)

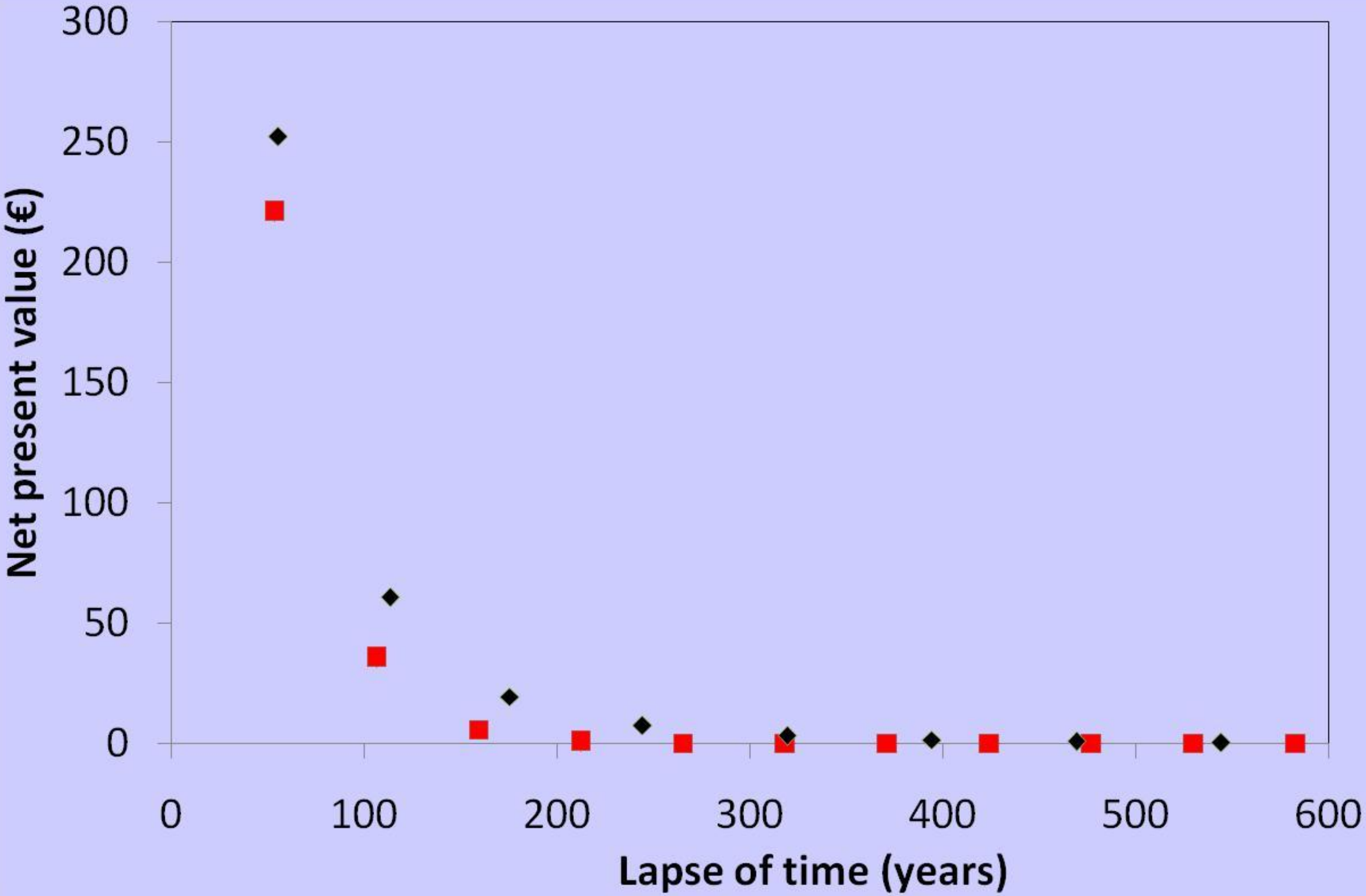


100
90
80
70
60
50
40
30
20
10
0
Rotation lengths (years)
0
100
200
300
400
500
600
Lapse of time (years)


7000
6000
5000
4000
3000
2000
1000
0
Revenue (€)
0
20
40
60
80
100
Rotation length (years)


60
50
40
30
20
10
0
Rotation lengths (years)
0
100
200
300
400
500
600
Lapse of time (years)

How serious is it?

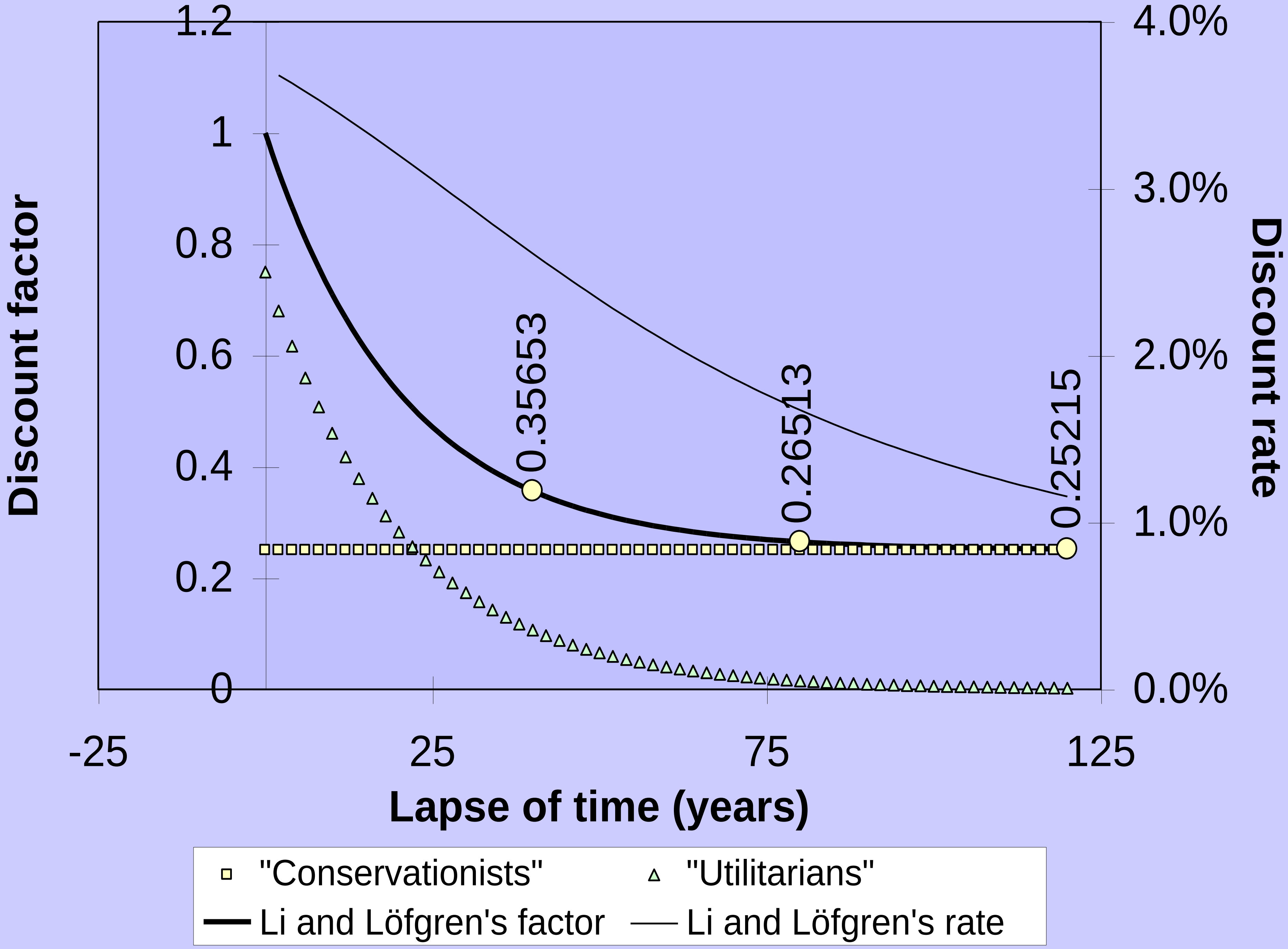


300
250
200
150
100
50
0
0
100
200
300
400
500
600
Net present value (€)
Lapse of time (years)

Reduced NPV from using the wrong criterion

		Criterion determining rotation length		
		NPV @ 1%	**NPV @ 3.5%**	**NPV declining**
Criterion of profitability	**NPV @ 1%**	€2274	€1972	€2118
	NPV @ 3.5%	€211	€264	€262
	NPV declining	€303	€341	€346

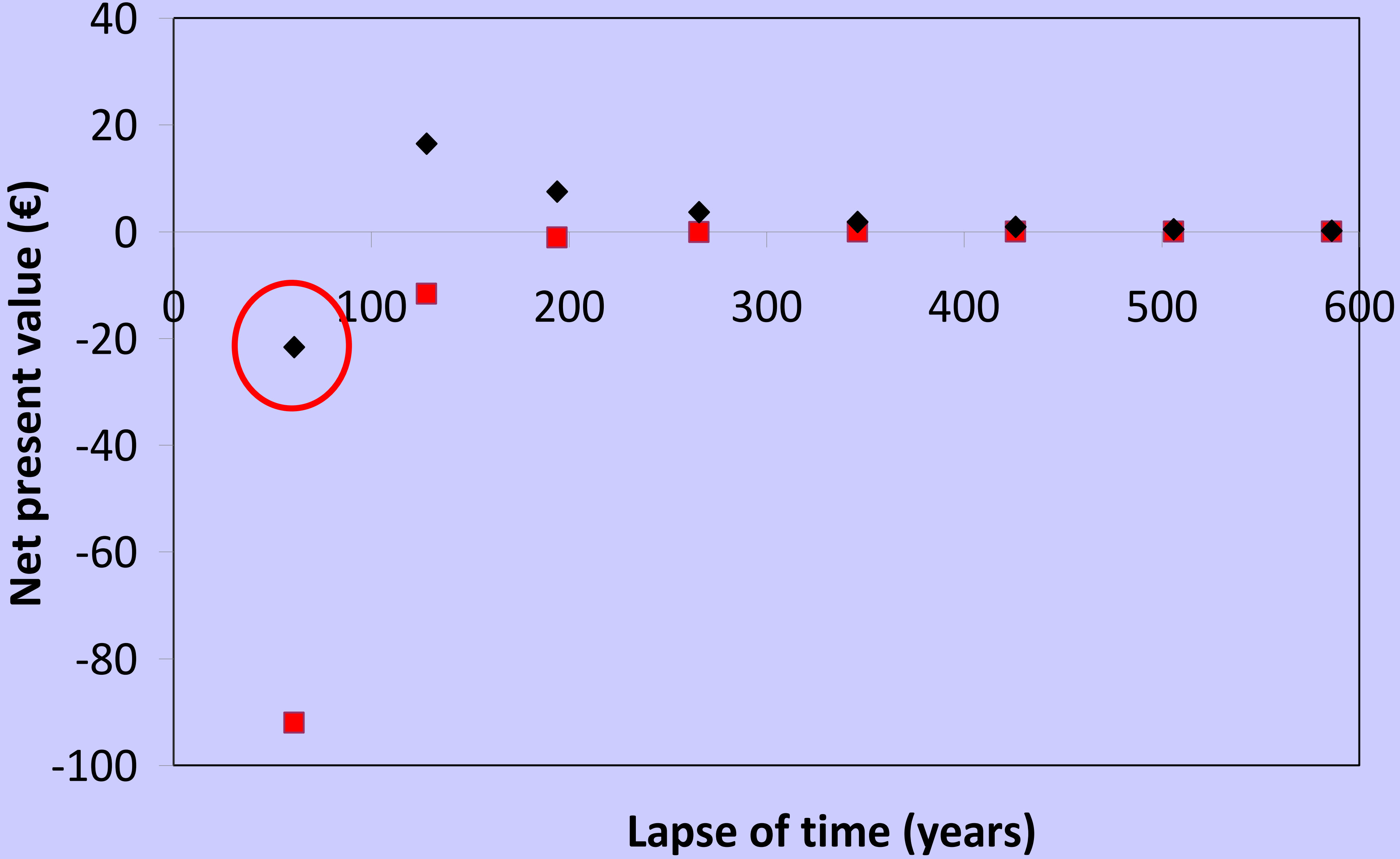
Dynamic inconsistency



Discount factor
Discount rate
1.2
1
0.8
0.6
0.4
0.2
0
4.0%
3.0%
2.0%
1.0%
0.0%
-25
25
75
125
0.35653
0.26513
0.25215
Lapse of time (years)
"Conservationists"
"Utilitarians"
Li and Löfgren's factor
Li and Löfgren's rate

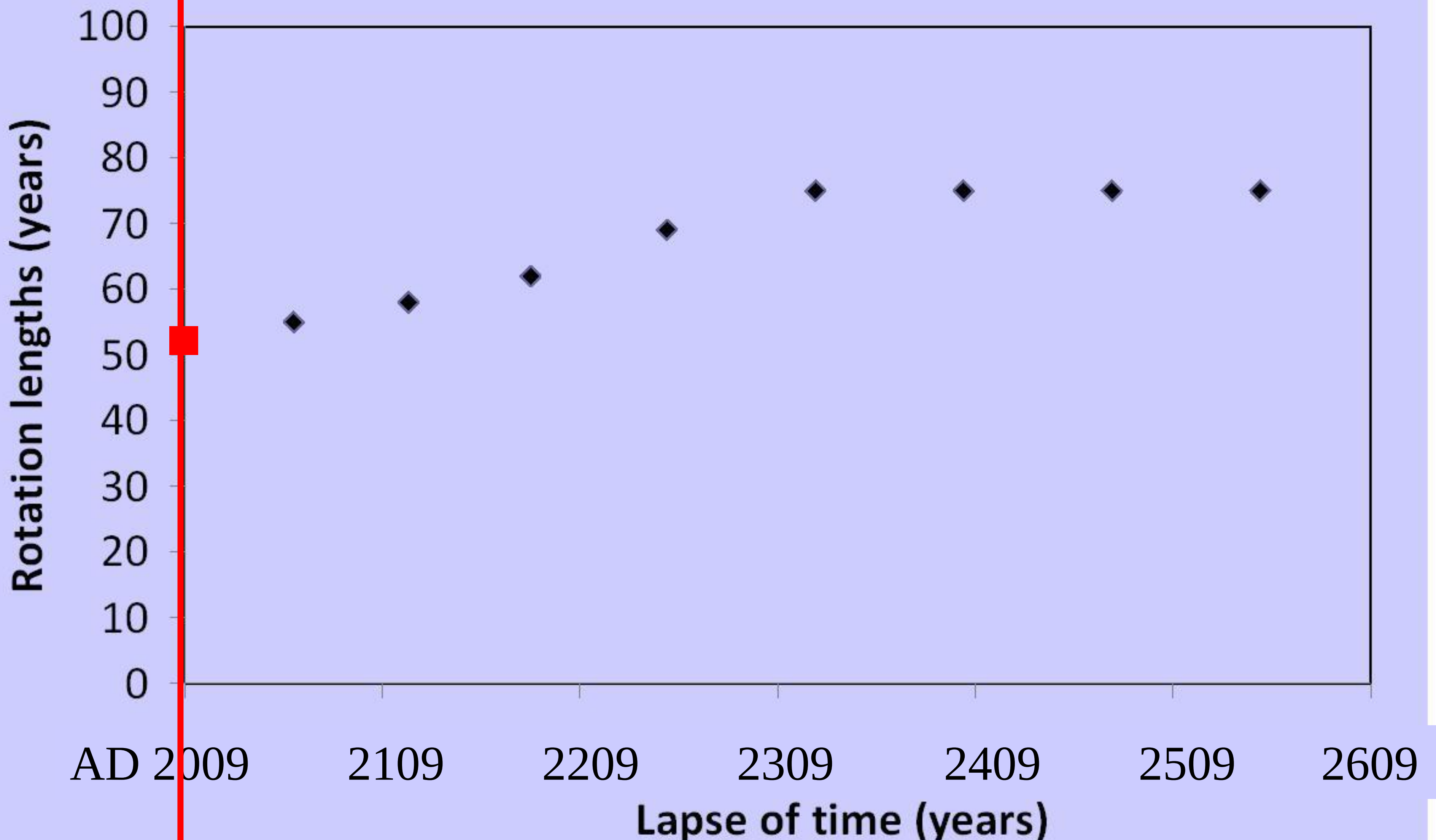
Event	Cash flow per hectare	Discounted value seen from time 2000	Discounted value seen from time 2080
Plant	–£2000	–£2000 × 1 = –£2000	
Fell at age 80	£6000	£6000 × **0.26613** = £1591	£6000 × 1 = £6000
Fell at age 120	£12000	£12 000 × **0.25215** = £3026	£12 000 × **0.35653** = £4278

Effect of heavy initial discounting with high establishment cost

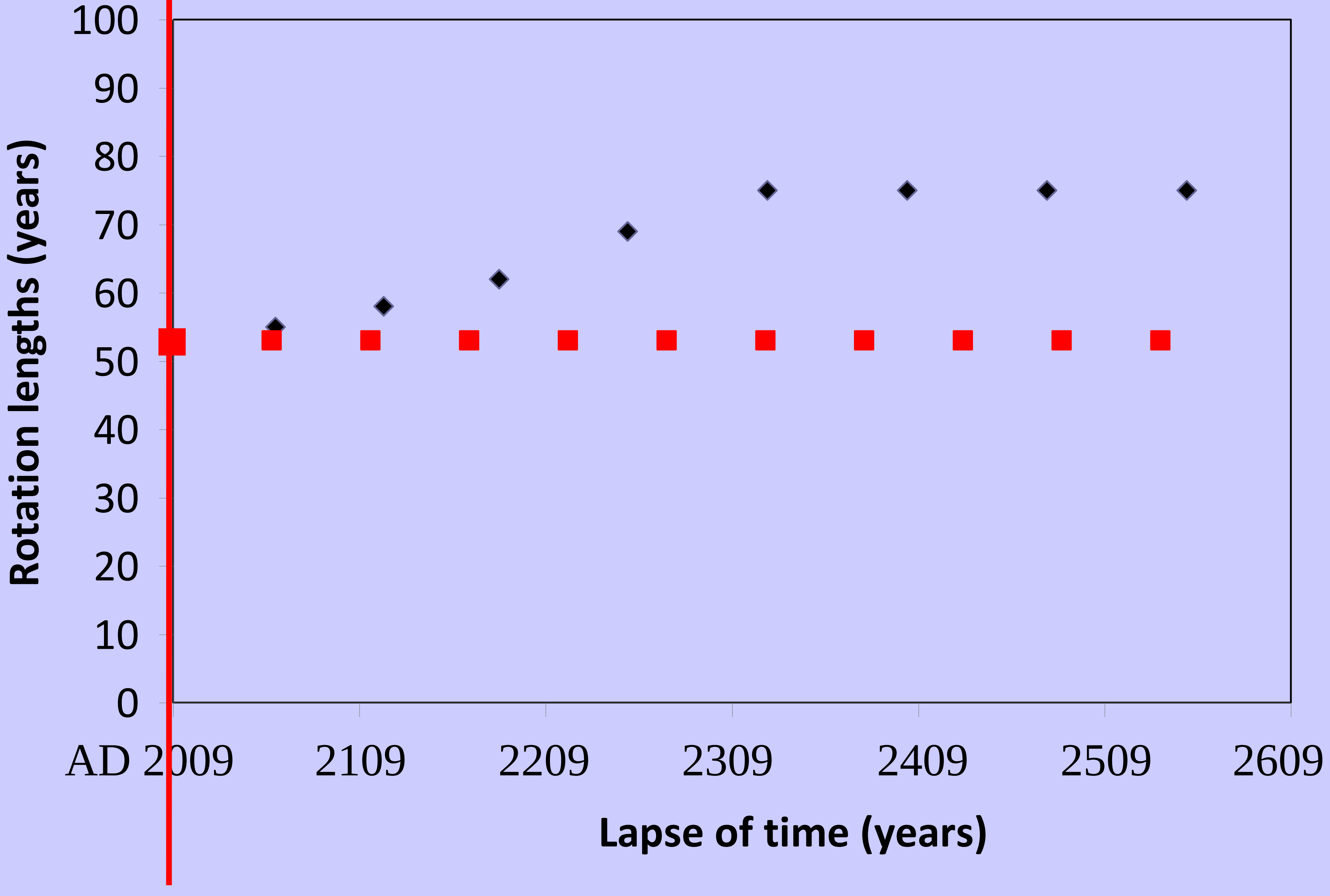


Dynamic consistency: shifting weight on objectives.

Naïve reassessment …



… and smart reassessment



Reduced NPV from using the wrong criterion

		Criterion determining rotation length		
		NPV @ 1%	**NPV @ 3.5%**	**NPV declining**
Criterion of profitability	**NPV @ 1%**	€2274	€1972	€2118
	NPV @ 3.5%	€211	€264	€262
	NPV declining	€303	€341	€346

Stabilisation problems yet to be resolved:

- ❖ dealing with stepped discount functions
- ❖ incorporating large formation costs
- ❖ adding an infinite series of rotations beyond 1000 years.

Not problems of constructing formulas, but ...
... of structuring calculation sequence.

Do the one from DXX3608 with multiple factors. Try in KulaLiLof in Excel|Livepaps|Discount? Just do a factor which is {(Kula+LiLof)/2+1}/2. Should be quick enough: copy graph and put on a further factor and discount rate.

How about the minimum compensation vs maximum endowment thing?