The Optimal Rotation Problem with Variable Forestland Holding Size

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The Faustmann Model

- The value of forest land - a sum of discounted net cash flow over an infinite period of time
- Land value maximization: optimal rotation age

$$\underset{T}{Max}\, LEV = \frac{V(T) - C}{e^{rT} - 1} - C$$

The Faustmann Model-Extension

- variable inputs and input costs
- non-timber outputs of a forest
- stochastic nature of output and input prices
- risk aversion by the producers
- uneven-aged forests

The Faustmann Model

limitations?

- a fixed area of land : Land – never been used as a choice variable, scale issue has not been addressed
- The cost of land manager has not been included: The existence of an entrepreneur and profit maximization – never been considered

The Faustmann Model

- Same treatment as Boulding (IRR)?
  - Internal rate of return maximizes value of capital, given land and labor;
  - Faustmann model maximizes land value given capital and labor;
  - Faustmann model makes the same mistake but from the capital owner's perspective.

Changing Timberland Ownership and Management Patterns

- Large-scale private forestland ownership in the United States has changed
- Buying and selling land becoming more and more easier, compared to earlier times
- Increasing number of small-scale forestland and non-traditional owners
- Institutional investment in forestry increasing – TIMOs, REITs – main objective is profit maximization

Profit Maximization

- Land area (A) is introduced as a choice variable in the decision making process of the forestland manager.
- Total timber volume (Q):

$$Q = Q(T, E, A); Q_T, Q_E, Q_A > 0; Q_{TT}, Q_{EE}, Q_{AA} < 0$$

  - T=rotation age
  - E=initial period silvicultural effort per acre

Profit Maximization

- The stumpage value (V) of a standing forest:

$$V = V(T, E, A) = pQ(T, E, A)$$

- Profit of the forest manager from one rotation:

$$\pi_1 = V(T, E, A)\, e^{-rT} - wEA - A\,R\, r^{-1}(1 - e^{-rT})$$

- Equivalent annual profit:

$$\pi = V(T, E, A)\, r(e^{rT} - 1)^{-1} - wEr(1 - e^{-rT})^{-1} - A\,R$$

- The profit maximization problem:

$$\underset{T,E,A}{Max}\, \pi = V(T, E, A)\, r(e^{rT} - 1)^{-1} - wE\, r(1 - e^{-rT})^{-1} - AR$$

Profit Maximization

- F.O.C. 1:

$$V_E(T, E, A)e^{-rT} = w$$

  - Initial period silvicultural effort per acre should be chosen at a level *E* such that the present value of the marginal increment in per acre timber value is equal to the per unit price of *E*.

Profit Maximization

- F.O.C. 2:

$$R = PV_A(T, E, A)r(e^{rT} - 1)^{-1}$$

  - Land area should be chosen at a level *A* such that the present value of the marginal increment in timber is equal to the per acre investment in *E* plus the land rent payment during one rotation.

Profit Maximization

- F.O.C. 3:

$$PV_T(T,E,A) = r[PV(T,E,A) - wE] + r[PV(T,E,A) - wE](e^{rT} - 1)^{-1}$$

  - Forest should be cut at age *T*, when the marginal increment to the value of trees (L.H.S.) equals to the sum of the sum of opportunity costs of investment tied up in the standing trees (1st part of R.H.S.) and in the firm (2nd part of R.H.S.).

Dynamic Conditions

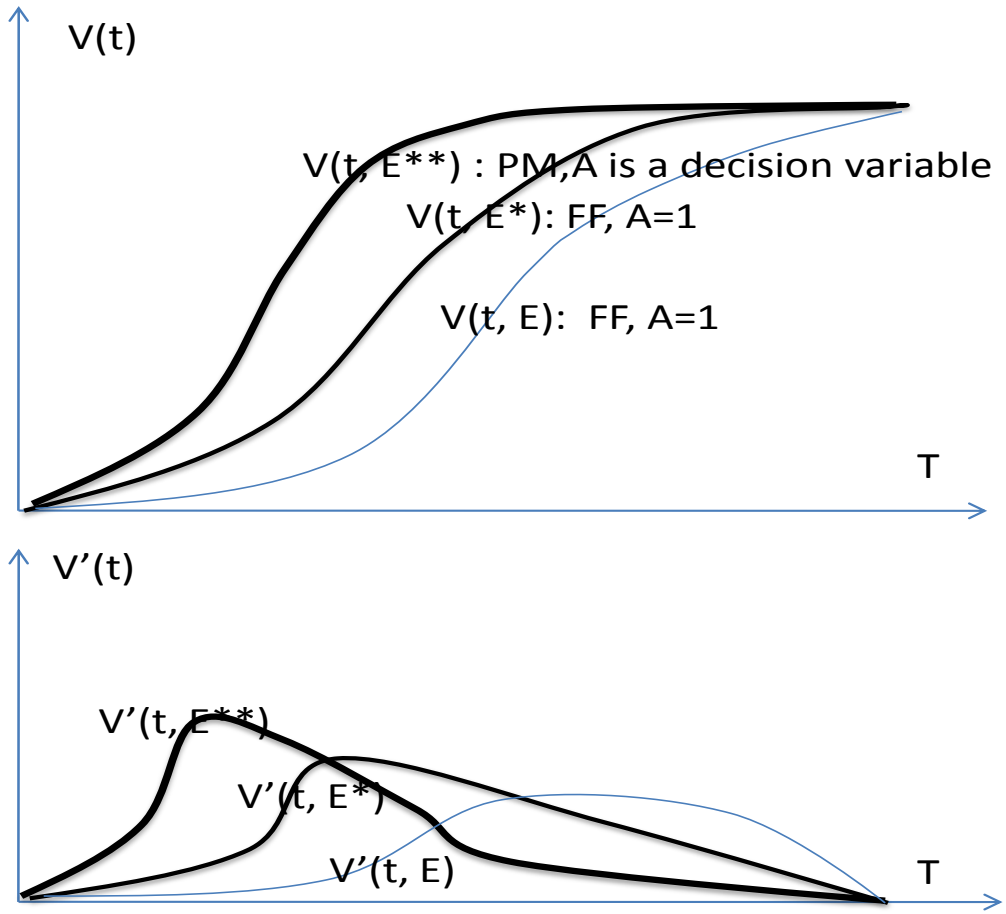
- Conceptual explanation:

$$Vevenue = pQ(A, E, T)$$

(R, W, i)

$$R,$$
$$rwE/(1-e^{-rT}),$$
$$[V_T(T,E,A)*V(T,E,A) - P*V(T,E,A) - R],$$
$$rPV(T,E,A)/(e^{-rT}-1).$$

Dynamic Conditions



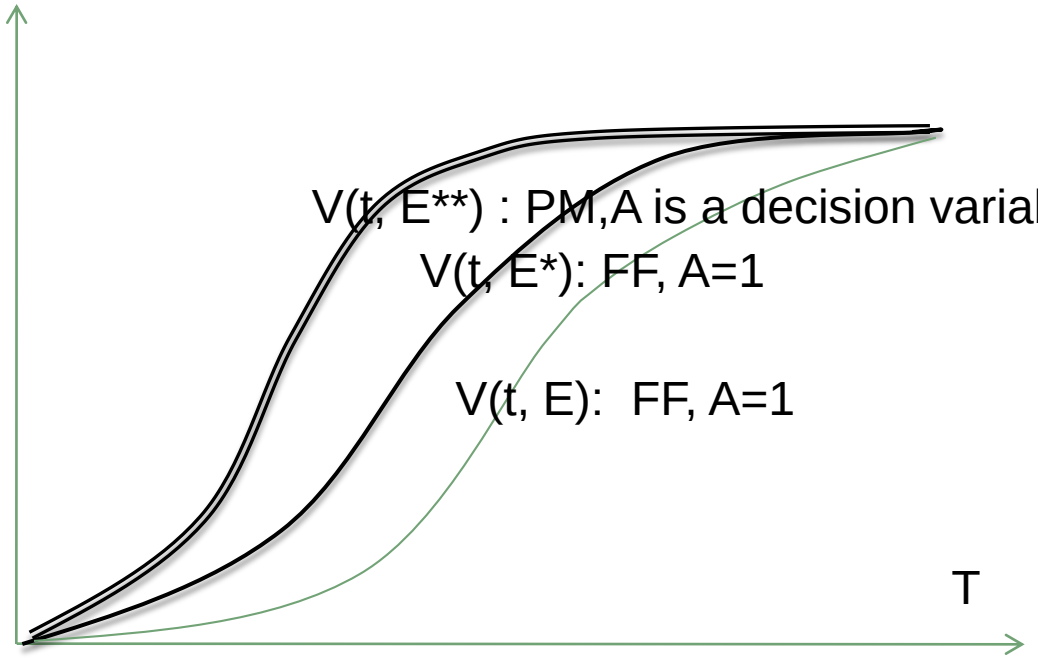
If w down:
A<1
E**>E*>E
T**<T*<T

If R up:
A<1
E**>E*>E
T**<T*<T

If P up:
A>1
E**<E*>E; E**>E
T**<T*>T; T**>T

If i up:
A<1
E**>E*<E; E**<E
T**>T*<T; T**<T

E** and E* are measured by input per unit of land, assuming that in the beginning it was 1 unit of land used



If w down:
A<1
E**>E*>E
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E**<E*>E; E**>E
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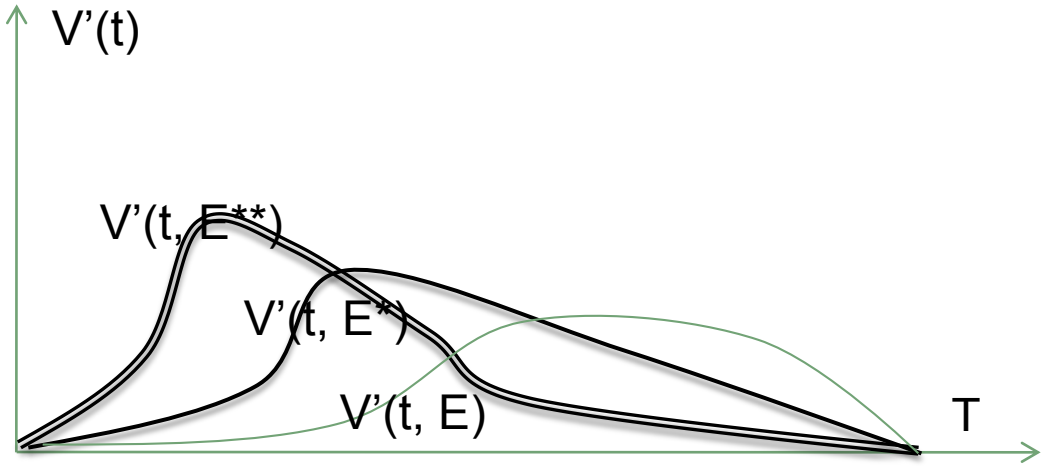

If i up:
A<1
E**>E*<E; E**<E
T**>T*<T; T**<T

E** and E* are measured by input per unit of land, assuming that in the beginning it was 1 unit of land used

Dynamic Conditions

- Silvicultural costs:
  - ↑ in w => ↓ in E
  - Additional land will be substituted for E
  - ↑ in T
  - ↓ in E/A (the input per unit of land) can be less than that in Faustmann model without the substitution effect => land will be less intensively managed
  - Therefore the impact on T would be larger than that in Faustmann model

Dynamic Conditions

- Interest rate:
  - ↑ in r => time becomes more costly => ↓ in T
  - ↑ in E, ↑ in A
  - Comparatively, impact on *E* will be larger than on *A* => land will be more intensively managed
  - Therefore, T will be shorter but longer than that in result in the Faustmann model.

Dynamic Conditions

- Land rent:
  - ↑ in R => ↓ in A => ↑ in E
  - Intensive management of land => ↑ in T
  - Changing E would affect the tree growth curve. Likely more intensive management would make trees grow faster and mature earlier => ↓ in T
  - Therefore, the final result on rotation is ambiguous, depending on the changed tree growth curve.

  - For LEV maximization, R is unknown. In fact, since it is essentially maximizing the annual rent, any assumed cost of land would not affect T.

Dynamic Conditions

- Stumpage price:
  - ↑ in p => ↑ in A, ↑ in E
  - ↑ in T
  - However, the substitution effect between A and E is uncertain.

Conclusions

- If land market is active, land under current management is constantly assessed by the owner against the market price.
- Capital, land and rotation are inter-dependent. We cannot address one factor without considering the other two.
- Profit maximization is more flexible and takes into consideration all factors —land, labor, capital, entrepreneur.
- Profit maximization is capable of considering both optimal rotation and optimal holding problems.

Thank You

Questions/Comments?