# Dynamic waves and the harvest of multiple rotations

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# Research Question

When should distant or less accessible but older stands be harvested rather than closer or more accessible but younger stands?

# Background/Context

European settlement of North America

Two geographical patterns

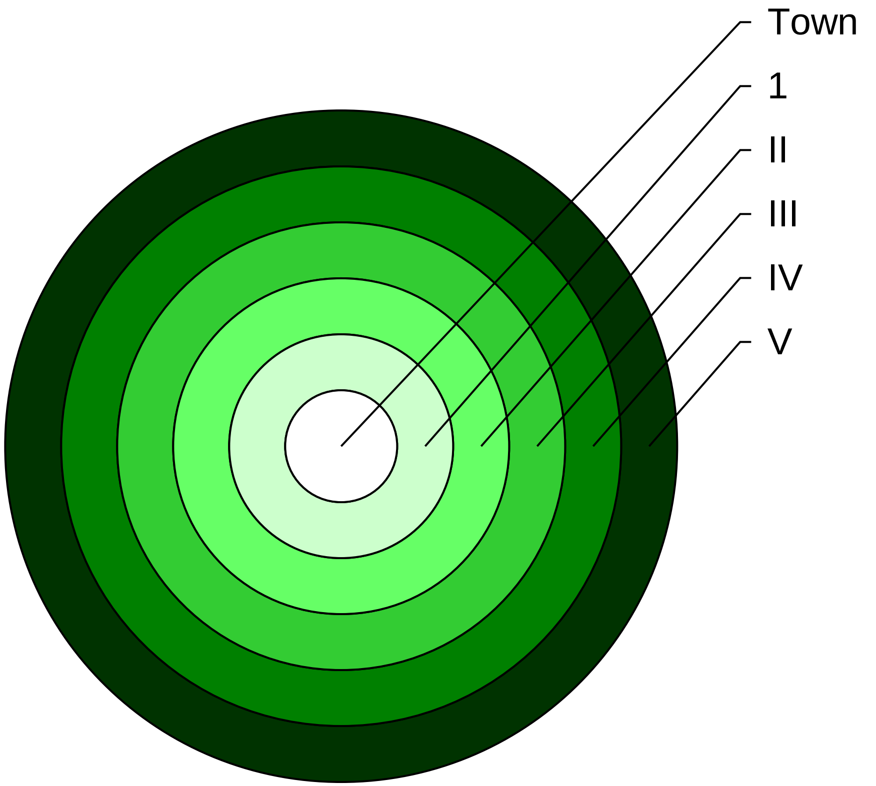
- Over time “old growth” forests harvested by region
- More accessible/closer stands harvested first

# Caveats

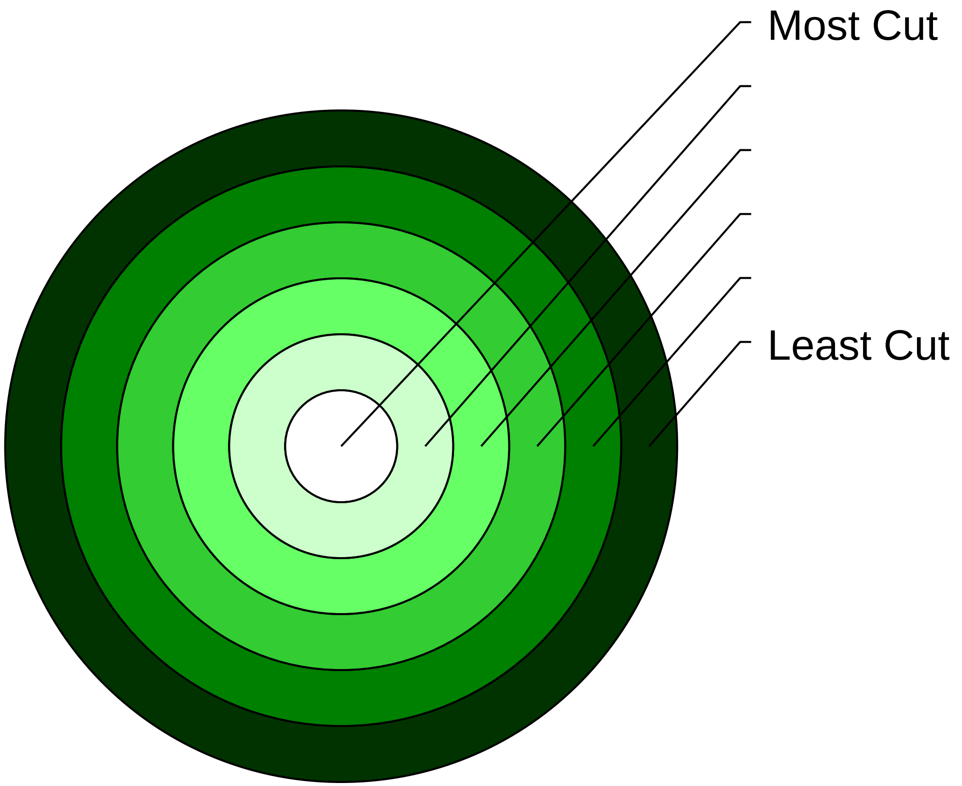
- Déjà vu
- Incomplete progress
- Lying: no numerical simulation
- Shading the truth:

  characterization of a derivation rather than derivation of results

# von-Thunen Rings



# Rotations



# Model Characterization

- Each rotation can be modeled as a depletion problem with stands/trees that grow
- At any time it may be optimal to harvest from more than one rotation
- Let a regime be the are harvest of some set of one or more rotations
- Regime switching problem

# Progress since 1999

- Some new results on multiple-phase optimal control problems: Doole 2008, Doole 2009 (switching conditions)

  (Older results: Amit 1986; Makris 2001)

- Development of notation (harvest ages for all hectares & rotations)

# Multiple-phase switching problem

- Implies that the transversality conditions for switching between harvesting regime j and harvesting regime j+1 are derivable

- Which implies that the transversality conditions for switching between harvesting rotation i and harvest rotation i+1 rotations are derivable

- Each rotation can be set up as a standard DP (or optimal control problem)

# Harvest order & notational simplification

From Hartwick 1977 it is easy to show:

- The order of hectares harvested is invariant over rotations
- The harvest of lower numbered rotations will start before the harvest of higher numbered rotations

# Harvest order, computational ease & notational simplification

- Determines harvest order within a rotation
- Limits the number of comparisons required at each time (number of rotations currently being harvested plus one)
- Allows harvesting regimes to be defined

# Objective Functional for each rotation

For every a and t,

$$V(X(t)) = \max_{H(a,t)} U\left(\sum_{a=0}^{a_{\max}} (P(a,t)Y(a) - C)H(a,t)\right) + BV(X(t+1)))$$

where

a is stumpage age,

$a_{max}$ is maximum harvest age,

B is the discount factor, i.e., B = (1+r)-1,

C is the fixed regeneration cost,

H(a,t) is hectares of age a stumpage harvested at time t,

P(a,t) is the per meter stumpage price for age a stumpage at time t,

X(t) is the vector of X(a,t)'s

Y(a) is the volume for age a stumpage; Y(.) is the volume function,

U(.) is the per period utility function, and

V(.) is the NPV from timber production given an initial vector of stock sizes X(t).

The objective is to maximize the sum of this period's utility and the discounted state of next period's stock.

# State Equations

For every t and a = 2 to $a_{max}$: $X(a+1,t+1) = X(a,t)-H(a,t,)$

For every t and a = 1: $X(1,t+1) = X_1$

For ages 2 to $a_{max}$, the number of hectares in age class a+1 at time t+1 is the number of hectares in age class a at time t minus the number of hectares harvested at time t+1

For age class 1, the number of hectares at time t+1 is the number of hectares harvested at time t

# Constraints

For every a and t, $0 \le H(a,t) \le X(a,t)$;

For every age and every time, harvest is non-negative and less than or equal to the stock.

# Multiple-phase solution

- Multiple-phase solution requires simultaneous determination of control variables and switching times
- Switching times are determined when setting the co-state variables of regime j and regime j+1 are equal

# Conclusion

- Multiple phase optimal control is applicable for a range of forest economics questions, specifically, when stands vary in age and spatial accessibility

- For some forest economics questions numerical simulation will be difficult because of the size of the decision space