

# **Did Pressler Understand how to Use the Indicator Per Cent?**

The third Faustmann Symposium

# Purposes

- To examine the relationship between Martin Faustmann's land expectation value formula and Max Pressler's Indicator Per Cent
- To appreciate some related works published in the 19<sup>th</sup> century by German scientists

# Introduction

- Wine-aging problem
  - Hartig (1833)
  - König (1835)
- Forest(land) valuation
  - Faustmann (1849a, 1849b, 1853)
- The indicator per cent
  - Pressler (1860a, 1860b, 1869)

# The Indicator Per Cent

- The average annual rate of increment of the capital of a forest stand
- The capital of a forest stand = stumpage value + land value
- Land value is constant
- Increment of stumpage value is decomposed into two parts
  - The first increment: growth of the timber stock
  - The second increment: increment of stumpage price

# Pressler (1860b)

## The first increment

$f(t)$  = the timber stock at age  $t$

The mean annual growth rate of the stand over a period of  $n$  years is

$$a = \left[ \frac{f(t+n)}{f(t)} \right]^{1/n} - 1$$

- The average annual increment is:

$$\frac{f(t+n) - f(t)}{n}$$

- The average timber stock is:

$$\frac{f(t+n) + f(t)}{2}$$

- The mean annual growth rate is approximately:

$$a = \frac{f(t+n) - f(t)}{f(t+n) + f(t)} \frac{200}{n}$$

# Pressler (1860b)

## The second increment (increment of stumpage price)

- Decrease in harvest cost
- Increase in the quality and dimension of the timber
- Change in timber prices (the third increment)

$P(t)$  = the stumpage price at age  $t$

The mean annual rate of increment of the stumpage price over a period of  $n$  years is:

$$b = \left[ \frac{P(t+n)}{P(t)} \right]^{1/n} - 1$$

The mean annual rate of increment is approximately:

$$b = \frac{P(t+n) - P(t)}{P(t+n) + P(t)} \frac{200}{n}$$

# Pressler (1860b)

## The value increment (stand value growth)

$$p = \frac{\Delta E(t)}{L + E(t)} = \frac{(a + b + ab)E(t)}{L + E(t)} \approx (a + b) \frac{R}{R + 1}$$

$E(t)$  = the stumpage value of the stand at age  $t$

$L$  = land value

$R = \frac{E(t)}{L}$  is the relative value of timber

$p$  is not the mean annual rate of value increase during years  $t$  to  $t + n$  if  $n$  is larger than 1, because  $R$  is not constant

# Pressler (1869)

The average annual increment of the capital of a stand is

$$w = \left( \frac{E(t+n) + L}{E(t) + L} \right)^{1/n} - 1$$

An approximation formula

$$w = \frac{E(t+n) - E(t)}{E(t+n) + E(t) + 2L} \frac{200}{n}$$

# Optimal Rotation Age

- Land expectation value maximization

$$\max_t L(t) = \frac{E(t) - Ce^{rt}}{e^{rt} - 1}$$

- The first-order condition:  $E'(T) = r[E(T) + L(T)]$

or

$$\frac{E'(T)}{E(T) + L(T)} = r$$



The marginal indicator per cent at age T

# Faustmann, Pressler and the optimal rotation age

- Faustmann formula
  - given a rotation age, what is the value of forestland?
- Pressler's Indicator Per Cent
  - what is the rate of return from a stand if it is left to grow for another  $n$  years?
- At the optimal rotation age, the marginal indicator per cent is equal to the interest rate
- The indicator per cent depends on land value
- To calculate the "correct" land value, we need to know the optimal rotation age

# Application of the Indicator Per Cent

- Pressler (1860b):  
one of the main objectives of economic silviculture is to attain the highest net revenue, which demands reaching and maintaining the highest indicator per cent of the timber.
  - *to establish the most valuable timber capital on the smallest possible land capital;*
  - *To try with all economic efforts conceivable to maintain the first as well as the second increment at the largest possible level.*
- von Seckendorf (1867)
- Judeich (1869)
- Kraft (1885)

# Conclusions

- Pressler was one of the few who understood in the 19th century that forestry decisions should be guided by the rate of increase of the capital and the interest rate instead of the timber growth rate
- Pressler's work focused on understanding and calculating the rate of return on forest capital (what he called the indicator per cent)
- How to use the indicator per cent in forestry decisions attracted little attention from Pressler
- The indicator per cent is a practically useful criterion for the decision on when to harvest stand, not for determining the theoretically correct optimal rotation age

Professor Dr. Alfred Clebsch (1833 - 1872)

Honorary Members of the London Mathematical Society



# Ueber ein Problem der Forstwissenschaft von Professor Dr. A. Clebsch zu Göttingen (1869 AFJZ sup pp. 1-16)

Daher ergibt sich zunächst für  $\phi(x, a)$  bei Wählerd:

$$24. \quad \phi(x, a) = 1 + \frac{\frac{x}{k} e^{-\frac{ax}{k}}}{1 - \frac{x}{k} e^{-ax}} - \frac{\frac{x^2}{k^2}}{1 - \frac{x}{k} e^{-ax}} \int_0^x e^{-\frac{t-x}{k}} \phi(t) dt - \frac{x}{1} \int_0^x e^{-\frac{t-x}{k}} \phi(t) dt$$

Die Funktion  $x_n$  bildet nach  $k^n$  in der Entwicklung dieses Ausdruck nach aufsteigenden Potenzen von  $x$ . Man kann jetzt zeigen, dass  $x_n$  zu berechnen, bei Wählerd  $\phi$  nach  $x^{n+1}$  entwickeln, mit  $k^n$  multipliciren und dann den Coefficienten von  $\frac{1}{x}$  ablesen. Und indem man dies nach der Entwicklung ausführt, findet man die Formel:

$$25. \quad x_n(x) = k_n \left[ \frac{e^{-\frac{ax}{k}}}{x^n (k - ax^{-k})} - \frac{1}{x^{n+1} (k - ax^{-k})} \int_0^x e^{-\frac{t-x}{k}} \phi(t) dt - \frac{1}{x^n} \int_0^x e^{-\frac{t-x}{k}} \phi(t) dt \right]_{x-1}$$

Speziellere ergibt sich daraus für  $a = 1$ :

$$x_1(x) = 1 - \frac{k}{1} \int_0^x \phi(t) dt.$$

Wie es sich zeigt, ist es aber auch leicht zu zeigen, dass

$$\begin{aligned} &= \left[ \frac{1}{k - ax^{-k}} \left\{ \frac{k^{n+1}}{x^{n+1}} e^{-\frac{ax}{k}} - \frac{k^n}{x^n} e^{-ax} + \frac{k^{n+1}}{x^{n+1}} \int_0^x e^{-\frac{t-x}{k}} dt \right\} \right]_{x-1} \\ &= \left[ \frac{1}{1 - ax^{-k}} \int_0^x \phi(t) dt \left\{ \frac{k^{n+1}}{x^{n+1}} e^{-\frac{t-x}{k}} - \frac{k^n}{x^n} e^{-\frac{t-x}{k}} + \frac{k^{n+1}}{x^{n+1}} \int_0^x e^{-\frac{t-x}{k}} dt \right\} \right]_{x-1} \\ &= \left[ \frac{k^{n+1}}{k^{n+1}} \int_0^x e^{-\frac{t-x}{k}} \phi(t) dt - \frac{k^n}{k^n} \int_0^x e^{-\frac{t-x}{k}} \phi(t) dt + \frac{k^{n+1}}{1^2 k^n} \int_0^x \left( \int_0^t e^{-\frac{t-u}{k}} \phi(u) dt \right) dx \right]_{x-1} \end{aligned}$$

Man kann zwei Entwicklungskoeffizienten, welche hier auf der rechten Seite stehen, jetzt hier nicht ablesen in

$$\left[ \frac{k^n}{x^{n+1}} \right]_{x-1}$$

Die Gleichung 25., welche die Functionen  $x_n$  (coefficiente) bestimmt, nach der Wählerd 25 befristet wird. Man hat nämlich aus 25.:

$$x_{n+1}(x) - x_n(0) + \frac{k}{1} \int_0^x x^n(x) dx =$$

aber, man offenbar befristet. Ebenso erfragt sich bei gewis Entwicklungskoeffizient auf

$$\left[ \frac{k^n}{k^n} \int_0^x e^{-\frac{t-x}{k}} \phi(t) dt \right]_{x-1}$$

wird sich gegen das gewisse Gleich bei dritten aufstellt. Die dritte befristet

$$x_{n+1}(x) - x_n(0) + \frac{k}{1} \int_0^x x_n(x) dx = - \left[ \frac{k^{n+1}}{k^{n+1}} \int_0^x e^{-\frac{t-x}{k}} \phi(t) dt + \frac{k^{n+1}}{1^2 k^n} \int_0^x \int_0^t e^{-\frac{t-u}{k}} \phi(u) dt dx \right]_{x-1}$$