

# **Faustmann Rotation and population dynamics in the presence of a risk of destructive events**

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For the management of natural resources, the first question that arises is : what is the optimal duration of cycle production. This is the case both in forestry, aquaculture, production of renewable resources. In the case where a calculation method to predict earnings for various terms of the cycle is available, Faustmann [2] proposed a formalism based on the expected discounted yield. Many authors have successively improved or reformulated the method, Ohlin [5], Pearse [6]. Clark [1] has applied this method to natural resources, in the absence of risk of destructive events. The risk of destruction has been introduced to forest stands by Martell [4] and Routledge [10] in discrete time. Thereafter, Reed [7] has studied the optimal forest rotation in continuous time with the risk of fire. Reed and Errico [8] have formulated and solved an optimal harvest scheduling problem in discrete time with the risk of fire using linear programming. Reed and Errico [9] have developed models to predict the long-run average yield in presence of a risk of infestation by pests and a risk of destructive fire. Stenger et al. [12] were interested in natural risks incurred by forests in discrete time. More recently Thorsen and Helles [13] maximize a not discounted criterion taking into account the risk and using a population model. In the context of random prices Guttormsen [3] studies a method based on dynamic programming. For the absence of risk of destructive events, all the production cycles are carried out to the same term. When the risk of destructive event exists and is taken into account, the authors cited above assume that the operator systematically decides to interrupt the current cycle and begin a new cycle. This is fully justified in the case of total destruction. In the case of a partially destructive event, to the first question about the optimal term a second question is added : should we interrupt the current cycle and begin a new cycle or is it better to continue the current cycle ? If there are alternatives, what is the criterion to choose ? To fulfill this goal in a framework of not too restrictive assumptions, first we define a criterion for choice and secondly we use a dynamic model population that allows us to follow the evolution of individuals of the system. To model population dynamics, contrary to those developed by Salo and Tahvonen [11] using age-structured models, we consider a simplified model of averaged individual type to facilitate the presentation of the proposed method and to focus the analysis on the influence of destructive event risk. The expected discounted yield is obtained via the resolution of a quasi-linear integrodifference equation.

For specific decisions and thus specific criteria, we study two particular cases. In the first case, the operator systematically interrupts the cycle in case of a destructive event, we generalize the results obtained by Reed [7] under less restrictive assumptions. We show that the results obtained in [7] are valid under the assumption that the operator does not harvest during the production cycle and that the cleaning costs in the case of destructive event be fixed. The possibility of taking into account intermediate harvesting and cleaning costs depending on the severity of damage therefore justifies the interest to introduce a model of population dynamics and to use the proposed method. This alone fully justifies the proposed approach. In the second case, the operator continues the cycle even in case of destructive events (which makes sense if the destruction is minor) and we deduce the corresponding expected discounted yield. Once the criterion set, in the general case for a test choice based on the number of individuals we show the existence of a unique solution to the integrodifference equation and we provide a numerical algorithm to solve it. Finally we show that the proposed formalism allows to integrate in a single optimization problem, the two levels of decision-making : the tactic level, with regard to harvesting (thinning in Forestry) and the strategic

level in case of destructive events with regard to the choice between two alternatives : to continue or to interrupt the cycle.

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